11. Vertical and horizontal sub-grid-scale transports*

11.1 Cumulus transport of momentum

The momentum change owing to the cumulus convection is divided into the following three components.

(1) The loss due to momentum entrainment into the cloud. These momentum are released from the cloud into the environment above that level.

(2) The gain due to momentum detrainment from the cloud.

(3) The momentum cahnge due to the cumulus subsidence.

The time rate of change in momentum due to cumulus convection is written as

$$\frac{\partial}{\partial t}\rho \mathbf{V} = -E\mathbf{V} + D\hat{\mathbf{V}} + \frac{\partial}{\partial z}(M_c\mathbf{V}), \tag{11.1}$$

where ρ is the density of the air, $\mathbf{V}$ is the horizontal velocity of the air in the environment, $\hat{\mathbf{V}}$ is the horizontal velocity of the air which is detraining from cumulus clouds, E is the total entrainment pet unit depth, D is the total detrainment per unit depth, and M_c is the total vertical mass flux of the clouds. E, D and M_c satisfy the mass budget equation,

$$-\frac{\partial M_c}{\partial z} + E - D = 0. \tag{11.2}$$

The total momentum is conserved by the redistribution of momentum due to cumulus convection. And

$$\int_{Z_B}^{Z_{max}} D(z)\, \hat{\mathbf{V}}(z)\, dz = \int_{Z_B}^{Z_{max}} E(z)\, \mathbf{V}(z)\, dz + \mathbf{V}_B\, M_c(z_B), \tag{11.3}$$

where z_B is the cloud base, z_{max} is the detrainment level of the deepest cloud, and $\mathbf{V}_B$ is the horizontal velocity of the air in the environment at z_B. See Chapter 7 and Fig. 11.1.

In a finite difference form, (11.1) at odd level k may be written as

$$\begin{aligned}\frac{\partial}{\partial t}\rho \mathbf{V}_k \Delta z_k = &-\sum_{i=kmin}^{k-2} \mathbf{V}_k m_{Bi} e_{k,i} \Delta z_k \\ &+ \sum_{k'=k+2}^{KN-1}{}' \mathbf{V}_{k'}\, m_{Bk} e_{k'},{}_k \Delta z_k + m_{Bk} \mathbf{V}_{KB} \\ &+ (M_c)_{k-1} \mathbf{V}_{k-i} - (M_c)_{k+1} \mathbf{V}_{k+1},\end{aligned} \tag{11.4}$$

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* This chapter is prepared by A. Kitoh.

where Σ' is the summation on odd levels only, i is the type of cloud which has the detrainment level at the odd level i (type-i cloud in Chapter 7), k_{min} is the index for the odd level at which the deepest clouds detrain, m_{Bi} is the sub-ensemble vertical mass flux at level KN, $e_{k,i}$ is the normalized entrainment of type-i cloud at level k, Δz_k is the depth of layer k, KN is the index of the even level immediately above the top of the planetary boundary layer, KB is the index of the odd level in which the top of the planetary boundary layer exists. $(M_c)_{k-1}$ is the total vertical mass flux at level k-1 and is given by

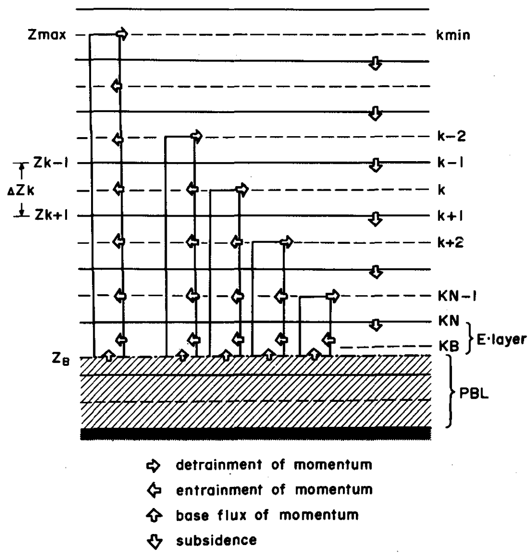

Fig. 11.1 Momentum budget of cumulus ensemble.

$$(M_c)_{k-1} = \sum_{i=kmin}^{k-2}{}' \eta_{k-1,i} M_{Bi}, \tag{11.5}$$

where η is the normalized mass flux of type-i cloud at level k. Entrainment and mass flux have the following relation

$$e_{k,i}\Delta z_k = \eta_{k-1,i} - \eta_{k+1,i}, \tag{11.6}$$

(see Fig. 11.2).

The first term on the right hand side of (11.4) represents the loss due to the momentum entrainment into the cloud which detrain above z_{k-1}. This term occurs because there is deeper cloud which has its top of the cloud above that level and this cloud entrains the environmental air into the cloud. The terms in the secend line represent the gain due to the momentum detrainment. The detrained air at level k consists of the entrained air into type-k cloud. These entrainment occurs at the odd levels below that level and at the top of the planetary boundary layer (cloud base flux). The third

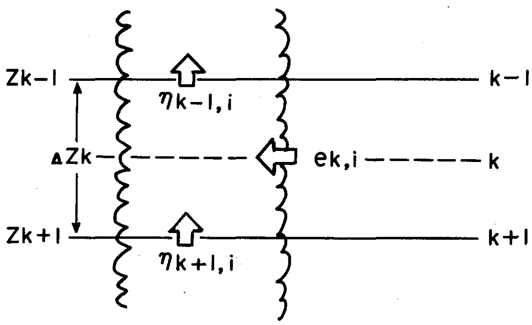

Fig. 11.2 Mass budget of type-i cloud at level k.

line represents the momentum change due to the cumulus subsidence.

11.2 Vertical diffusion

Currently no vertical diffusions are included in any equations.

11.3 Horizontal Diffusion

In the model horizontal sub-grid-scale transport is introduced in momentum equations in terms of the horizontal non-linear eddy diffusion described in Holloway and Manabe (1971). The horizontal diffusion terms are expressed as follws ;

$$\frac{\partial}{\partial t}(\Pi^{(u)}_{i+\frac{1}{2},j} u_{i+\frac{1}{2},j}) + \cdots\cdots = (\frac{m}{\Delta\xi})_j\ (\tau^{\lambda\lambda}_{i+1,j} - \tau^{\lambda\lambda}_{i,j}) + (\frac{nm^2}{\Delta\eta})_j \left\{ (\frac{\tau^{\lambda\varphi}}{m^2})_{i+\frac{1}{2},j+\frac{1}{2}} - (\frac{\tau^{\lambda\varphi}}{m^2})_{i+\frac{1}{2},j-\frac{1}{2}} \right\} \quad (11.7)$$

$$\frac{\partial}{\partial t}(\Pi^{(v)}_{i,j+\frac{1}{2}} v_{i,j+\frac{1}{2}}) + \cdots\cdots = (\frac{m}{\Delta\xi})_{j+\frac{1}{2}} (\tau^{\lambda\varphi}_{i+\frac{1}{2},j+\frac{1}{2}} - \tau^{\lambda\varphi}_{i-\frac{1}{2},j+\frac{1}{2}}) - (\frac{nm^2}{\Delta\eta})_{j+\frac{1}{2}} \left\{ (\frac{\tau^{\lambda\lambda}}{m^2})_{i,j+1} - (\frac{\tau^{\lambda\lambda}}{m^2})_{i,j} \right\} \quad (11.8)$$

$$\tau^{\lambda\lambda}_{i,j} = \frac{1}{2}\left\{ (\Pi K_H)_{i+\frac{1}{2},j} + (\Pi K_H)_{i-\frac{1}{2},j} \right\} \cdot (D_T)_{i,j} \quad (11.9)$$

$$(D_T)_{i,j} = (\frac{m}{\Delta\xi})_j (u_{i+\frac{1}{2},j} - u_{i-\frac{1}{2},j}) - (\frac{n}{m\Delta\eta})_j \left\{ (vm)_{i,j+\frac{1}{2}} - (vm)_{i,j-\frac{1}{2}} \right\} \quad (11.10)$$

$$\tau^{\lambda\varphi}_{i+\frac{1}{2},j+\frac{1}{2}} = \frac{1}{2}\left\{ (\Pi K_H)_{i+\frac{1}{2},j+1} + (\Pi K_H)_{i+\frac{1}{2},j} \right\} \cdot (D_S)_{i+\frac{1}{2},j+\frac{1}{2}} \quad (11.11)$$

$$(D_S)_{i+\frac{1}{2},j+\frac{1}{2}} = (\frac{m}{\Delta\xi})_{j+\frac{1}{2}} (v_{i+1,j+\frac{1}{2}} - v_{i,j+\frac{1}{2}}) + (\frac{n}{m\Delta\eta})_{j+\frac{1}{2}} \left\{ (um)_{i+\frac{1}{2},j+1} - (um)_{i+\frac{1}{2},j} \right\} \quad (11.12)$$

$$(K_H)_{i+\frac{1}{2},j} = (\frac{\Delta\xi\Delta\eta}{mn})_j \cdot \min\left[k_0^2 \left[\frac{1}{2}\left\{ (D_T)^2_{i,j} + (D_T)^2_{i+1,j} + (D_S)^2_{i+\frac{1}{2},j+\frac{1}{2}} + (D_S)^2_{i+\frac{1}{2},j-\frac{1}{2}} \right\}\right]^{\frac{1}{2}}, D_{comax} \right] \quad (11.13)$$

The notations used in the above are the same as those used in Chapter 3. The reason for keeping K_H less than $D_{comax}(\frac{\Delta\xi\Delta\eta}{mn})$ is to avoid linear instability due to diffusion terms, and D_{comax} is set to $1/\left\{4(\Delta t)_d \cdot (\frac{m}{\Delta\xi}\frac{\Delta\eta}{n})_j\right\}$, where $(\Delta t)_d$ is the time interval of evaluating diffusion terms (see Fig. 5.1). The constant k_0 is assigned the value 0.2.

In other equations, *i.e.* the thermodynamic equation, moisture equation and ozone equation, no horizontal diffusion terms are included.