Vertical Layer Placement in the Eta Coordinate for Models with a High Model Top

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1. Background

The vertical coordinate system used in the operational global model of the JMA (Japan Meteorological Agency); i.e., the GSM (Global Spectral Model), is the Eta (σ-p hybrid) coordinate (Simmons and Burridge 1981, Simmons and Strüfing 1983). The JMA is now planning to increase the number of vertical layers in this model from the current 60 to 100 levels, and to lift the model top full level from 0.1 to 0.01 hPa in the near future. In addition, flexible change of the model's vertical levels is often required for various research activities and numerical experiments; e.g., an increase in the number of layers around the tropopause for research related to troposphere–stratosphere interaction, or an increase in the number of layers in the boundary layer for studies of boundary layer clouds. However, there are few papers related to the determination of vertical levels (e.g. Eckermann 2009).

Regarding the determination of vertical levels, it is an extremely important issue at what altitude more layers should be distributed in numerical weather prediction or climate models. However, this depends on what phenomena we wish to simulate the most realistically, what score we wish to improve the most, and to what extent each physical parameterization (boundary layer, gravity wave, convection, etc.) can take advantage of the fine vertical levels. Another issue is how to obtain a smooth profile of the vertical layer distribution after deciding on the approximate weights for the layer distribution. In this report, the latter issue is discussed.

There are two possible methods to determine the placement of the layers in a vertical coordinate system. One is the connection of some mathematical functions that retains the continuities up to the *n*-th order derivatives. This method can certainly provide a smooth profile of the depth of the layers; however, it is difficult when using this method to meet very detailed requirements for the distribution of the layers to each altitude, to save computing resources, and maximize the total model performance. The second method is to obtain a smooth function after arbitrarily giving some pairs of (k, $p_{k-1/2}$) for an assumed surface pressure p_s, where $p_{k-1/2}$ are half-level pressures for the k-th model level (k = 1 for the bottom layer). However, in this case it is not so easy to obtain a smooth function, due both to the wide pressure range (five orders of magnitude: 1000–0.01 hPa), and also because accurate fitting is required, not only near the top, but also near the surface where model vertical resolution is much higher than at the mid-level around 800–300 hPa. If we use a simple logarithm of pressure for the fitting, the deviation will be significant for the levels near the surface. Therefore, a practical method to obtain a smooth and satisfactory fitting function for the layer placement is considered here.

2. Method

Half-level pressures $p_{k-1/2}$ can be written using the constants $A_{k-1/2}$ and $B_{k-1/2}$ (k = 1, 2, …, $k_{\max}$) in the Eta coordinate as follows:

$$p_{k-1/2} = A_{k-1/2} + B_{k-1/2} p_s \quad (1)$$

where $k_{\max}$ is the total number of layers. The continuous functions $p(\tilde{k})$, $A(\tilde{k})$, and $B(\tilde{k})$, where $\tilde{k}$ is real and corresponds to integer values of k, are defined according to discrete value sets of $p_{k-1/2}$, $A_{k-1/2}$, and $B_{k-1/2}$. In the following discussion, $p_s = p_{1000} \equiv$ 1000 hPa is assumed, and a function $\mu(p)$ is defined as a weight of $A(\tilde{k})$ in $p(\tilde{k})$:

$$p(\tilde{k}) = A(\tilde{k}) + B(\tilde{k}) p_{1000} \quad (2)$$

$$\mu(p) \equiv A(\tilde{k}) / p(\tilde{k}) \quad (3)$$

In 1999, T. Matsumura (JMA) developed a method in which $p_{k-1/2}$ are determined first, and then $A_{k-1/2}$ and $B_{k-1/2}$ are determined using a given function $\mu(p)$. The same procedure is also adopted here.

2.1. Determination of $p_{k-1/2}$

[Step 1] First, a fitting polynomial of degree eight $f(\tilde{k})$ that fits the data (k_n, $\log(p_{kn-1/2})$) corresponding to the arbitrarily given pairs (k_n, $p_{k\,n-1/2}$) (n = 1, 2, …, N. 8 < N ≤ $k_{\max}$.) is calculated using the least squares method:

$$f(\tilde{k}) = \log(p_s) + \sum_{i=1}^{8} e_i \left(\tilde{k} - 1\right)^i \quad (4)$$

where e_i are fitting parameters. When the function $f(\tilde{k})$ is fitted, the weight for each given data point is extremely important. If the assumed errors for the fitting are the same for all given data, it is obvious that the data around the model top cannot be fitted. In contrast, if the assumed errors are proportional to $p_{kn-1/2}$, the obtained function significantly deviates from the given data near the surface. Therefore, $\Delta p_{kn-1/2}$ are assumed as the relative magnitude of the errors ($\Delta p_{kn-1/2}$ / $p_{kn-1/2}$ for logarithm). By using this weight, the function can fit the given data well, both near the model top and the surface. Although we cannot determine the exact values of $\Delta p_{kn-1/2}$ at this stage, we can use rough approximations calculated from the given data. If more accurate values are required, they can be obtained by iteration.

[Step 2] Although the function $f(\tilde{k})$ is obtained in Step 1, it cannot be guaranteed that the layer thickness profile is sufficiently smooth. Therefore, a smooth function $g(\tilde{k})$, which corresponds to $\Delta\log(p)$, is calculated as follows. A polynomial $g(\tilde{k})$ of degree six, which has a lower degree than $f(\tilde{k})$, is used to obtain an adequately smooth profile of layer thickness:

$$g(\tilde{k}) = \log(2) + \sum_{i=1}^{6} d_i \left(\tilde{k} - (k_{\max} - 1)\right)^i \qquad (5)$$

where d_i are fitting parameters. The data $\Delta\log(p_{k-1/2})$ for $k = 1, 2, \ldots, k_{\max-1}$ are calculated using eq. (4) as input data for the fitting. The relative errors in the fitting are assumed to be proportional to $\Delta\log(p_{k-1/2})$. The first term in eq. (5) means that $\Delta p_{k-1/2}$ for the second layer from the top is equal to that of the top layer.

[Step 3] As the layer depth profile is fitted in Step 2, the sum of the layer depth $\Delta p_{k\,n-1/2}$ is not consistent with p_s. Therefore, the function obtained in Step 2, $g(\tilde{k})$, is normalized as follows:

$$\alpha = \{\log(p_s) - \log(p_{k\max-1/2})\} / \left[\sum_{k=1}^{k\max-1} \{\Delta\log(p_{k-1/2})\}\right] \qquad (6)$$

$$h(\tilde{k}) = \alpha g(\tilde{k}) \qquad (7)$$

[Step 4] Finally, the half-level pressure $p_{k-1/2}$ can be obtained as follows:

$$\log(p_{k-1/2}) = \log(p_s) - \sum_{k'=1}^{k} h(k') \qquad (8)$$

An example of the difference profiles of the obtained $p_{k-1/2}$ for 100 vertical levels (Fig. 1) shows that this method can generate a smooth profile for the vertical layers, which is created from the fitting of arbitrarily given data.

2.2. Determination of $A_{k-1/2}$ and $B_{k-1/2}$

An Eta coordinate that approaches the isobaric coordinate upwards faster than the operational Eta coordinate is required for performance tests related to the change of layer distribution. A new function of $\mu(p)$, which is a combination of two functions and has the following characteristics, is created. The pressures where $\mu(p)$ first becomes 1 and 0 are $p_{\min}$ and $p_{\max}$ for each, and the pressure where $\mu(p) = 0.5$ is p_{cnt}. This means that the coordinate is a completely isobaric coordinate above $p_{\min}$, and a σ-coordinate below $p_{\max}$. Continuities up to the second-order derivatives are imposed at $\mu(p) = 0.5$ (for a variable $\log(p)$). The first derivatives of $\mu(p)$ at $p_{\min}$ and $p_{\max}$ are also set to zero. The function is as follows:

$$\mu(p) = \begin{cases} 1 + \sum_{i=2}^{3} a_i \left(\log(p) - \log(p_{\min})\right)^i & 0.5 \le \mu \le 1 \\ 0 + \sum_{i=2}^{3} b_i \left(\log(p) - \log(p_{\max})\right)^i & 0 \le \mu \le 0.5 \end{cases} \qquad (9)$$

$$\begin{aligned} a_2 &= 3(2c_1c_2 + c_1^2 - c_2^2) / \{4c_1^2 c_2 (c_2 - c_1)\} \\ a_3 &= (-4c_1c_2 - 3c_1^2 + c_2^2) / \{4c_1^3 c_2 (c_2 - c_1)\} \\ b_2 &= 3(2c_1c_2 - c_1^2 + c_2^2) / \{4c_1 c_2^2 (c_2 - c_1)\} \\ b_3 &= (-4c_1c_2 + c_1^2 - 3c_2^2) / \{4c_1 c_2^3 (c_2 - c_1)\} \end{aligned} \qquad (10)$$

$$\begin{aligned} c_1 &= \log(p_{cnt}) - \log(p_{\min}) \\ c_2 &= \log(p_{cnt}) - \log(p_{\max}) \end{aligned} \qquad (11)$$

Fig. 2 shows the created version of function $\mu(p)$, where $p_{\min}$ = 60 hPa, $p_{\max}$ = 1000 hPa, and p_{cnt} = 400 hPa are assumed, together with the operationally used function. The figure shows that in the case of the created function, the Eta coordinate smoothly approaches the isobaric coordinate in the upper troposphere faster than the operational Eta coordinate.

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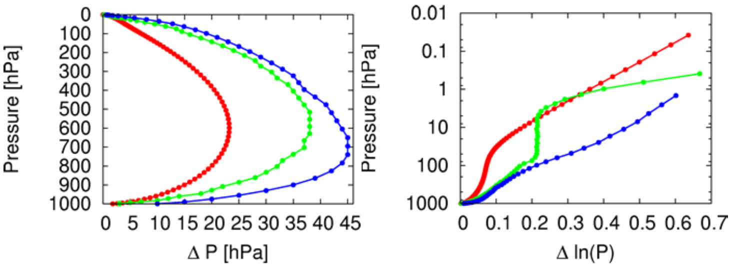

Fig. 1: Vertical layer placement. An example of GSM L100 created by the procedure described in the text (red), GSM L60 in operation (green), and GSM L40 which was in operation until 2007. Profiles of $\Delta p_{k\,n-1/2}$ (left) and $\Delta\log(p_{k-1/2})$ (right). Note that $\Delta\log(p_{k-1/2})$ at the top is infinity.

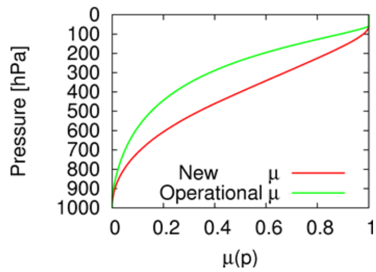

Fig. 2: Profile of $\mu(p)$ (a weight of coefficient of A in the pressure p for layers, where p_s = 1000 hPa is assumed). The new function (red) and the function used in the operational model (green).