F. Boundary Conditions

F-1. Boundary conditions for the pressure equation

F-1-1 E-HI-VI scheme

In section D-3, we discussed the pressure solver for the Neumann boundary conditions (D3-2-4) to (D3-2-6). The definite expressions of B_x, B_y and B_z were obtained by computing the pressure gradient terms in the relevant momentum equations.

For the E-HI-VI scheme, the lateral boundary conditions are given from (C3-1-10) and (C3-1-11) as

$$\frac{\partial \Delta^2 P}{\partial x} = -\frac{\Delta^2 U}{(1+\alpha)\Delta t} - 2ADVU'$$

$$= -2\{ADVU' + \frac{\partial U}{\partial t} - \frac{U^{it} - U^{it-1}}{(1+\alpha)\Delta t}\}, \qquad \text{(F1-1-1)}$$

$$\frac{\partial \Delta^2 P}{\partial y} = -\frac{\Delta^2 V}{(1+\alpha)\Delta t} - 2ADVV'$$

$$= -2\{ADVV' + \frac{\partial V}{\partial t} - \frac{V^{it} - V^{it-1}}{(1+\alpha)\Delta t}\}. \qquad \text{(F1-1-2)}$$

The right-hand sides of the above equations include the time tendency of wind speed at the boundaries. Since the modified advection terms ($ADVU'$; $ADVV'$, ..) defined in (C3-1-14) and (C3-1-15) include surface friction and Coriolis forces as in (C2-1-20) and (C2-1-21), the model gives a kinematically balanced pressure field as the solution. We will refer to the time tendency of horizontal winds again in section F-2-2.

The upper boundary condition from (C3-1-12) is

$$\left(\frac{\partial}{\partial z^*} + \frac{1}{\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}}\overline{\overline{\frac{g}{mC_m{}^2}}}\right)\Delta^2 P = \left(-\frac{\Delta^2 W}{(1+\alpha)\Delta t} - 2ADVW'\right)/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}$$

$$= -2\{ADVW' + \frac{\partial W}{\partial t} - \frac{W^{it} - W^{it-1}}{(1+\alpha)\Delta t}\}/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}$$

$$= -2\{ADVW' + \frac{EXT.W - W^{it-1}}{2\Delta t} - \frac{W^{it} - W^{it-1}}{(1+\alpha)\Delta t}\}/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}. \qquad \text{(F1-1-3)}$$

Here, $EXT.W$ is the expected value of W at the upper boundary, which is currently set to zero.

The lower boundary condition is given by setting

$$G^{\frac{1}{2}}\Delta^2 W^* = \Delta^2 W + m(G^{\frac{1}{2}}G^{13}\Delta^2 U + G^{\frac{1}{2}}G^{23}\Delta^2 V) = 0, \qquad \text{(F1-1-4)}$$

and substituting it into (F1-1-3) with (C3-1-10) and (C3-1-11) as

$$\left(\frac{\partial}{\partial z^*} + \frac{1}{\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}}\overline{\overline{\frac{g}{mC_m{}^2}}}\right)\Delta^2 P = \{-2ADVW' - 2m(G^{\frac{1}{2}}G^{13}ADVU' + G^{\frac{1}{2}}G^{23}ADVV')\}/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}$$

$$\underline{-mG^{\frac{1}{2}}G^{13}\frac{\partial \Delta^2 P}{\partial x} + G^{\frac{1}{2}}G^{23}\frac{\partial \Delta^2 P}{\partial y})/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}}. \qquad \text{(F1-1-5)}$$

An iterative procedure is necessary to solve (D3-1-1), since the under-lined term in the above equation is not separable. However, if the model includes surface friction and we assume $W=0$ at the ground surface, we can use

$$(\frac{\partial}{\partial z^*}+\frac{1}{\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}}\overline{\overline{\frac{g}{mC_m{}^2}}})\Delta^2 P=-2ADVW'/\overline{\overline{\frac{1}{mG^{\frac{1}{2}}}}}, \tag{F1-1-6}$$

for the lower boundary condition instead of (F1-1-5), and then the iterative procedure is no longer required.

P.G. Lateral boundary conditions for pressure equations are set in the array PFORCE in sub.CFPBDV.

F-1-2 AE scheme

The boundary conditions for an anelastic model are the same as described by Ikawa and Saito (1991). The lateral boundary conditions are, from (C2-2-4) and (C2-2-5),

$$\frac{\partial P}{\partial x}=-\frac{\partial U}{\partial t}-\frac{\partial G^{\frac{1}{2}}G^{13}P}{G^{\frac{1}{2}}\partial z^*}-ADVU+RU, \tag{F1-2-1}$$

$$\frac{\partial P}{\partial y}=-\frac{\partial V}{\partial t}-\frac{\partial G^{\frac{1}{2}}G^{23}P}{G^{\frac{1}{2}}\partial z^*}-ADVV+RV, \tag{F1-2-2}$$

where the right-hand sides of the above equations include the time tendency of wind speed at the boundaries, as well as surface friction and Coriolis forces as the forcing functions. The second terms on the right-hand side come from the residual terms of the horizontal pressure gradient forces in the chain rule. The inner values of P just adjacent to the boundaries at the former time step are used in these terms.

The upper boundary condition is obtained from (C2-2-9) as

$$(\frac{\partial}{\partial z^*}+\frac{gG^{\frac{1}{2}}}{C_s{}^2})P=G^{\frac{1}{2}}(-\frac{\partial W}{\partial t}-ADVW+BUOY'+RW)$$
$$=G^{\frac{1}{2}}(-\frac{EXT.W-W^{it-1}}{2\Delta t}-ADVW+BUOY'+RW), \tag{F1-2-3}$$

where $EXT.W$ is currently set to zero.

The lower boundary condition is given by setting

$$G^{\frac{1}{2}}\frac{\partial W^*}{\partial t}=\frac{\partial W}{\partial t}+G^{\frac{1}{2}}G^{13}\frac{\partial U}{\partial t}+G^{\frac{1}{2}}G^{23}\frac{\partial V}{\partial t}=0, \tag{F1-2-4}$$

and substituting it into (F1-2-3) with (C2-2-4) and (C2-2-5) as

$$(\frac{\partial}{\partial z^*}+\frac{gG^{\frac{1}{2}}}{C_s{}^2})P=G^{\frac{1}{2}}\{G^{\frac{1}{2}}G^{13}(-\frac{\partial p}{\partial x}-\frac{\partial G^{\frac{1}{2}}G^{13}P}{G^{\frac{1}{2}}\partial z^*}-ADVU+RU)$$
$$+G^{\frac{1}{2}}G^{23}(-\frac{\partial p}{\partial y}-\frac{\partial G^{\frac{1}{2}}G^{23}P}{G^{\frac{1}{2}}\partial z^*}-ADVV+RV)-ADVW+BUOY'+RW\}. \tag{F1-2-5}$$

In order to maintain separability, the above equation is arranged as

$$(\frac{\partial}{\partial z^*}+\overline{\overline{\frac{gG^{\frac{1}{2}}}{C_s{}^2}}})P=G^{\frac{1}{2}}\{BUOY'-ADVW+RW-G^{\frac{1}{2}}G^{13}(ADVU-RU)-G^{\frac{1}{2}}G^{23}(ADVV-RV)\}$$
$$\underline{+(\overline{\overline{\frac{gG^{\frac{1}{2}}}{C_s{}^2}}})-\frac{gG^{\frac{1}{2}}}{C_s{}^2})P-G^{\frac{1}{2}}\{G^{\frac{1}{2}}G^{13}(\frac{\partial P}{\partial x}+\frac{\partial G^{\frac{1}{2}}G^{23}P}{G^{\frac{1}{2}}\partial z^*})+G^{\frac{1}{2}}G^{23}(\frac{\partial P}{\partial y}+\frac{\partial G^{\frac{1}{2}}G^{23}P}{G^{\frac{1}{2}}\partial z^*})\}}, \tag{F1-2-6}$$

where the underlined terms are the variable part in the iterative application of the direct method.

P.G. Lateral boundary conditions for pressure equations are set in the array PFORCE in sub.CFPBDV.

F-2. Lateral boundary conditions

F-2-1 Cyclic boundary condition

As described in B-7-1 of Ikawa and Saito (1991), the cyclic boundary conditions are available in x- and/or y- directions. For the x-direction cyclic boundary condition,

$$\phi_{1,,}=\phi_{nx-1,,},$$
$$\phi_{nx,,}=\phi_{2,,}, \qquad \text{(F2-1-1)}$$

are imposed for all field variables ϕ. Similar conditions are applied to the y-direction for the y-direction cyclic condition.

As described in B-7-2 of Ikawa and Saito (1991), the free-slip wall lateral boundary condition is an option.

P.G. The cyclic condition is employed when MSWSYS(14)=1 or 2 ; the wall lateral boundary condition is used when MSWSYS(14)=-1 or -2. See sub.LTRLB2, LTRLBU, LTRLBV, LTRLUV and ADJ2D1.

F-2-2 Open boundary condition

a) Normal wind

Orlanski's (1976) radiation condition is used for wind normal to the lateral boundaries. According to the Sommerfeld radiation condition, the phase velocity of the inner gravity wave is evaluated by

$$C^{\tau}_{\phi_b}=-\frac{\phi_b{}^{\tau+1}-\phi_b{}^{\tau-1}}{2\Delta t}\Big/\frac{\dfrac{\phi_b{}^{\tau+1}+\phi_b{}^{\tau-1}}{2}-\phi^{\tau}_{in}}{\Delta x}$$
$$=-\frac{\phi_b{}^{\tau+1}-\phi_b{}^{\tau-1}}{\phi_b{}^{\tau+1}+\phi_b{}^{\tau-1}-2\phi_{in}{}^{\tau}}\frac{\Delta x}{\Delta t}. \qquad \text{(F2-2-1)}$$

Here, the subscripts b and in indicate variables at the boundary and inner adjacent points. This phase velocity C is replaced by the smoothed phase velocity C^* that is averaged at three inner grid points at the former time step, and we choose the smaller value compared to $\Delta x/\Delta t$ as

$$C_{\phi}{}^*=\min\left(\frac{1}{3}\sum_{i=1}^{3}C^{\tau-1}_{\phi_{b-i}},\frac{\Delta x}{\Delta t}\right). \qquad \text{(F2-2-2)}$$

The radiative value ϕ_{RAD} at the outflow boundary is then given as

$$\phi_{RAD}=\frac{1-\dfrac{\Delta t}{\Delta x}C_{\phi}{}^*}{1+\dfrac{\Delta t}{\Delta x}C_{\phi}{}^*}\phi^{\tau-1}_{in}+\frac{2\dfrac{\Delta t}{\Delta x}C_{\phi}{}^*}{1+\dfrac{\Delta t}{\Delta x}C_{\phi}{}^*}\phi^{t}_{in} \quad for\ C_{\phi}{}^*>0. \qquad \text{(F2-2-3)}$$

At the inflow boundary, ϕ_{RAD} is assumed to be the same as the boundary value at the former time step :

$$\phi_{RAD}=\phi_b{}^{\tau-1} \qquad for\ C_{\phi}{}^*\leq 0. \qquad \text{(F2-2-4)}$$

The value at the boundary is finally determined by the following weighted averaging procedure using ϕ_{EXT} and ϕ_{RAD} as

$$\phi^{\tau+1}_{BND}=\alpha\phi_{EXT}+(1-\alpha)\phi_{RAD}, \qquad \text{(F2-2-5)}$$

where ϕ_{EXT} is the external reference value, and α is a weighting parameter.

Three values of α,

α_{in} ; $for\ U < 0\ and\ C_\phi^* \leq 0,$

α_{out1} ; $for\ U \geq 0\ and\ C_\phi^* \leq 0,$

α_{out2} ; $for\quad C_\phi^* > 0,$ (F2-2-6)

are employed depending on the directions of the normal wind and gravity waves. In the nesting case, these values should be determined according to the reliability of the prediction of the outer model. If maximum values such as $\alpha_{in}=\alpha_{out1}=\alpha_{out2}=1$ are used, the lateral boundary values of the nested model are strictly determined by the outer model only, and the radiative condition becomes meaningless. If the smallest values, such as $\alpha_{in}=\alpha_{out1}=\alpha_{out2}=0$, are used, the lateral boundary values are determined only by the radiation condition, which means that the inner model can no longer incorporate the time change of the environmental field from the outer model. In most nested runs, a value greater than 0.5 is used for α_{in}, while relatively smaller values are employed for α_{out1} and α_{out2}.

Using (F2-2-5), the time tendency of the normal wind component at the lateral boundary is *tentatively* given as

$$\left(\frac{\partial\phi}{\partial t}\right)_b^{\tau *}=\frac{\phi_{BND}^{\tau+1}-\phi_b^{\tau-1}}{2\Delta t}. \qquad \text{(F2-2-7)}$$

This tentative time tendency of the normal wind is modified so that it guarantees the conservation of the total mass in the model domain as discussed in F-2-3.

b) for other variables

For variables other than the wind component normal to the lateral boundaries, gradient extrapolation is usually used at the outflow boundary. The inflow and outflow are distinguished by the direction of the normal wind component:

$$\phi_{RAD}=2\phi_{in}^{\tau}-\phi_{in-1}^{\tau-1}\quad for\ U\geq 0, \qquad \text{(F2-2-8)}$$

$$\phi_{RAD}=\phi_{out}^{\tau-1}\qquad for\ U<0. \qquad \text{(F2-2-9)}$$

Here, ϕ means the prognostic variables of the model such as the wind component parallel to the lateral boundaries, potential temperature, and water vapor mixing ratio. Since their locations are placed on a staggered grid (Fig. D1-1-2), subscripts *out* and *in* mean the external and internal values just adjacent to the lateral boundary. The external value just adjacent to the lateral boundary at the next time level is determined using a weighted averaging procedure similar to (F2-2-5):

$$\phi_{out}^{\tau+1}=\beta\phi_{EXT}+(1-\beta)\,\phi_{RAD}, \qquad \text{(F2-2-10)}$$

Three values of β are available, depending on the directions of wind and gravity waves, similar to (F2-2-6), though usually β_{out2} is set equal to β_{out1}. The potential temperature is extrapolated using the deviation from the reference potential temperature as

$$\theta_{out}^{\tau+1}=\beta_{out}\theta_{EXT}+(1-\beta_{out})\,[\overline{\theta}+\{2(\overline{\theta}-\theta_{in}^{\tau})-(\overline{\theta}-\theta_{in-1}^{\tau-1})\}], \qquad \text{(F2-2-11)}$$

to prevent artificial buoyancy, considering the steepness of the orography at the boundary.

P.G. Normal wind is extrapolated in sub.EXTNUH and sub.EXTNVH ; tangential wind is extrapolated in sub. EXTUY2 and sub.EXTVX2. Other variables are extrapolated in sub.EXTRX1, EXTRY1, EXTRX2 and

EXTRY2.

F-2-3 Mass flux adjustment for radiative nesting

The relation between the time tendency of the total mass within the entire model domain and the total mass flux through the lateral boundaries (E2-1-1) must be satisfied not only for the initial conditions but also for the entire simulation period. Although U_0' and V_0' in (E2-1-2) satisfy (E2-1-1) for each output time of the outer model, the normal wind components determined by the radiation condition (F2-2-5) are not necessarily consistent with (E2-1-1). A simple adjustment scheme for an anelastic model was given in Ikawa and Saito (1991). In a fully compressible nesting model, the time tendency of the normal wind should be modified so that the mass flux at each model boundary conforms to the mass flux of the outer model as closely as possible while satisfying (E2-1-1).

For the given adjusted interpolated horizontal wind (*e.g.,* $U_0'(,,,KT)$ and $U_0'(,,,KT+\Delta KT)$) of the outer model, the external wind at t and its tendency between KT and $KT+\Delta KT$ are given by

$$U_{EXT}(,,,t) = U_0'(,,,KT) + (t-KT)\frac{\partial}{\partial t}U_{EXT}, \qquad \text{(F2-3-1)}$$

$$\frac{\partial}{\partial t}U_{EXT}=\frac{U_0'(,,,KT+\Delta KT)-U_0'(,,,KT)}{\Delta KT}. \qquad \text{(F2-3-2)}$$

The time tendency of the mass-flux through each lateral plane that satisfies the total mass conservation is obtained as follows:

$$\frac{\partial MFX1}{\partial t}=\frac{\partial}{\partial t}\left(\iint U_{EXT}dydz^*\right)_{x=0}$$
$$=\frac{\iint U_0'(2,,,KT+\Delta KT)\,dydz^*-\iint U_0'(2,,,KT)\,dydz^*}{\Delta KT}, \qquad \text{(F2-3-1)}$$

$$\frac{\partial MFX2}{\partial t}=\frac{\partial}{\partial t}\left(\iint U_{EXT}dydz^*\right)_{x=X}$$
$$=\frac{\iint U_0'(nx,,,KT+\Delta KT)\,dydz^*-\iint U_0'(nx,,,KT)\,dydz^*}{\Delta KT}, \qquad \text{(F2-3-2)}$$

$$\frac{\partial MFY1}{\partial t}=\frac{\partial}{\partial t}\left(\iint V_{EXT}dxdz^*\right)_{y=0}$$
$$=\frac{\iint V_0'(,2,,KT+\Delta KT)\,dxdz^*-\iint V_0'(,2,,KT)\,dxdz^*}{\Delta KT}, \qquad \text{(F2-3-3)}$$

$$\frac{\partial MFY2}{\partial t}=\frac{\partial}{\partial t}\left(\iint V_{EXT}dxdz^*\right)_{y=Y}$$
$$=\frac{\iint V_0'(,ny,,KT+\Delta KT)\,dxdz^*-\iint V_0'(,ny,,KT)\,dxdz^*}{\Delta KT}. \qquad \text{(F2-3-4)}$$

Finally, the time tendencies of the normal wind component in each model plane given by (F2-2-7) are adjusted by

$$\left(\frac{\partial U}{\partial t}\right)_0^\tau=\left(\frac{\partial U}{\partial t}\right)_0^{\tau*}+\left\{\frac{\partial MFX1}{\partial t}-\int_s\left(\frac{\partial U}{\partial t}\right)_0^{\tau*}dydz^*\right)\}/S_{yz}, \qquad \text{(F2-3-5)}$$

$$\left(\frac{\partial U}{\partial t}\right)_{X_L}^\tau=\left(\frac{\partial U}{\partial t}\right)_{X_L}^{\tau*}+\left\{\frac{\partial MFX2}{\partial t}-\int_s\left(\frac{\partial U}{\partial t}\right)_{X_L}^{\tau*}dydz^*\right)\}/S_{yz}, \qquad \text{(F2-3-6)}$$

$$\left(\frac{\partial V}{\partial t}\right)_0^\tau=\left(\frac{\partial U}{\partial t}\right)_0^{\tau*}+\left\{\frac{\partial MFY1}{\partial t}-\int_s\left(\frac{\partial V}{\partial t}\right)_0^{\tau*}dxdz^*\right)\}/S_{xz}, \qquad \text{(F2-3-7)}$$

$$\left(\frac{\partial V}{\partial t}\right)^{\tau}_{Y_L}=\left(\frac{\partial V}{\partial t}\right)^{\tau}_{Y_L}{}^{*}+\left\{\frac{\partial MFY2}{\partial t}-\int_s\left(\frac{\partial V}{\partial t}\right)^{\tau}_{Y_L}{}^{*}dxdz^{*}\right)\right\}/S_{yz}, \tag{F2-3-8}$$

where S_{yz} and S_{xz} are the areas of the lateral plane of the model at the east-west and north-south lateral boundaries, respectively. The above wind speed adjustment is only order of 10^{-4} m/s^2 in most cases, but is crucial for maintaining a reasonable average pressure field of the nested model. The above time tendencies of normal winds are used for the Neumann-type boundary conditions for pressure diagnostic equations (F1-1-1) and (F1-1-2). For an AE scheme, similar procedures are done assuming the left-hand side of (E2-1-1) as zero, and the time tendencies are used for the first terms of *r.h.s.* in (F1-2-1) and (F1-2-2).

P.G. The time tendency of the mass-flux through each model plane is computed in sub.CDMFDT2. The time tendency is adjusted in sub.UVPBD.

F-2-4 Boundary relaxation

Rayleigh damping, which enforces the external values of the prognostic variables, can be imposed near the lateral boundaries. In Ikawa and Saito's (1991) model, the external values were fixed to those of a one-dimensional reference atmosphere. In the new model, time-dependent three-dimensional values interpolated from the outer model are used for the boundary relaxation.

Rayleigh damping

$$D_R=-\frac{1}{\Delta t}\frac{D_{xy}}{m_g}\{\phi(x,y,z,t)-\phi_{EXT}(x,y,z,t)\}, \tag{F2-4-1}$$

is added to the time tendency of ϕ, where m_R is the coefficient that determines the $1/e$-folding time. D_{xy} is a function given by

$$D(x,x_d,X_L)=\frac{x_d-x}{x_d} \quad for\ x<x_d,$$

$$D(x,x_d,X_L)=\frac{x-(X_L-x_d)}{x_d} \quad for\ x>X_L-x_d, \tag{F2-4-2}$$

$$D_{xy}(x,y,x_d)=\max\{D(x,x_d,X_L),\ D(x,x_d,Y_L)\}. \tag{F2-4-3}$$

Here x_d is the width of the sponge layer where the boundary relaxation is enforced. At $x=0$ and $x=X_L$, the $1/e$-folding time of (F2-4-1) becomes $m_R\Delta t$.

P.G. Parameters m_R and x_d are given by RLDMPX and IDIFX in the parameter card. D_{xy} is set in sub.SETDCF and used in sub.RLDAMP3.

F-3. Upper and lower boundary conditions

F-3-1 Velocity and potential temperature

From (D2-1-12), the kinematic condition

$$W^{*}=\frac{1}{G^{\frac{1}{2}}}\{W+m(\overline{G^{\frac{1}{2}}G^{13}\overline{U}^{z}}^{x}+\overline{G^{\frac{1}{2}}G^{23}\overline{V}^{z}}^{y})\}=0, \tag{F3-1-1}$$

is imposed for W at $k=1+1/2$ and $k=nz-1/2$. For U and V, a free-slip condition is imposed for the upper boundary and lower boundary, if there is no friction. Under non-slip conditions, sub-grid scale momentum fluxes are given from the resistance law as in B-10-2 of Ikawa and Saito (1991).

F-3-2 Absorbing layer

Rayleigh damping, which enforces the external values to the prognostic variables, is imposed near the upper boundary. In Ikawa and Saito's model (1991), the external values were fixed to those of a one-dimensional reference atmosphere. In the new model, time-dependent three-dimensional values given by the outer model are available for nesting.

Rayleigh damping

$$D_{Rz} = -\frac{1}{\Delta t}\frac{D_z}{m_{Rz}}\{\phi(x,y,z,t) - \phi_{EXT}(x,y,z,t)\}, \qquad \text{(F3-2-1)}$$

is added to the time tendency of ϕ, where m_{Rz} is the coefficient that determines the e-folding time. D_z is a function given by

$$D_z(z,z_d,H) = \frac{1+\cos(\frac{H-z}{H-z_d}\pi)}{2} \quad for\ z > z_d. \qquad \text{(F3-2-2)}$$

Here, x_d is the width of the sponge layer where the boundary relaxation is enforced. At $z=H$, the e-folding time of (F3-2-1) becomes $m_{Rz}\Delta t$.

P.G. Parameters m_{Rz} and z_d are given by RLDMPZ and KZDST in the parameter card. D_z is set in sub.SETDCF and used in sub.RLDAMP3.