E. Initiation of the model

E-1. Reference atmosphere and initial environmental field

E-1-1 Stand-alone case

For the stand-alone case (without nesting), the input parameter card specifies vertical profiles of horizontal wind (u, v), potential temperature (θ) and mixing ratio of water vapor (q_v) (see Section K). In Ikawa and Saito (1991), the reference atmosphere was given by a simple algebraic function, while in the new model, the vertical profile given by the input parameter card is used for the reference atmosphere without any deformation except vertical interpolation into the model levels. For given θ and q_v, the pressure and density of the reference atmosphere are given by

$$\overline{\theta_v}=\overline{\theta}(1+0.61\overline{q_v}), \tag{E1-1-1}$$

$$\overline{\pi_g}=\left(\frac{\overline{p_s}}{p_0}\right)^{R/C_p}, \tag{E1-1-2}$$

$$\overline{\pi}=\overline{\pi_g}-\frac{g}{C_p}\int_0^z\frac{1}{\overline{\theta_v}}dz, \tag{E1-1-3}$$

$$\overline{p}=p_0\overline{\pi}^{C_p/R}, \tag{E1-1-4}$$

$$\overline{\rho}=\frac{p_0}{R\overline{\theta_v}}\left(\frac{\overline{p}}{p_0}\right)^{C_v/C_p}, \tag{E1-1-5}$$

where $\overline{p_s}$ is the averaged pressure at $z=0$. Once $\bar{\rho}(z)$ is obtained, the reference atmosphere is interpolated on the z^* coordinate

$$\overline{\theta_v}(z)\rightarrow\overline{\theta_v}(x,y,z^*),$$
$$\overline{p}(z)\rightarrow\overline{p}(x,y,z^*),$$
$$\overline{\rho}(z)\rightarrow\overline{\rho}(x,y,z^*), \tag{E1-1-6}$$

In order to satisfy $DIVT(U, V, W)=0$ and $W^*=0$ in entire model domains, the initial momentum as the prognostic variables U, V and W is set by

$$U(z^*)=\overline{\rho}(z)u(z),$$
$$V(z^*)=\overline{\rho}(z)v(z), \tag{E1-1-7}$$
$$W(x,y,z^*)=-(G^{\frac{1}{2}}G^{13}U+G^{\frac{1}{2}}G^{23}V). \tag{E1-1-8}$$

Here we assumed $m=1$ for the stand-alone case. Note that, U and V in (E1-1-7) differ from the alternative expression by their original definition:

$$U(x,y,z^*)=\rho(x,y,z^*)G^{\frac{1}{2}}u(z),$$
$$V(x,y,z^*)=\rho(x,y,z^*)G^{\frac{1}{2}}v(z), \tag{E1-1-9}$$

Expression (E1-1-7) gives constant values for U and V at the z^* surface, that is, greater values than (E1-1-9) are used for the initial horizontal wind components if there are mountains. This modification is important for achieving a smooth start-up.

E-1-2 Nested case

When the non-hydrostatic model is used as the nested model, initial fields are prepared by interpolating the field of the outer model. As described in Saito (1994), two different interpolation procedures are employed. For the horizontal wind and the specific humidity, vertical interpolation is performed using the z^* coordinate in each model so that the boundary layer structure of the outer model is retained in the nested model. For the potential temperature, vertical interpolation is performed using the z coordinate to prevent artificial buoyancy due to the difference in the orographic heights between the models.

Once the specific humidity and potential temperature are interpolated, their horizontal average is computed to make the reference atmosphere:

$$\overline{\theta}(x,y,z^*) \rightarrow \overline{\theta}(z),$$
$$\overline{q_v}(x,y,z^*) \rightarrow \overline{q_v}(z), \qquad \text{(E1-2-1)}$$

where averaging is performed strictly on the horizontal coordinate, not on the terrain-following model coordinate. Once θ and q_v are computed, the reference atmosphere is set by (E1-1-1) to (E1-1-5), and interpolated into the model planes by (E1-1-6). Pressure and density at the initial field at each grid point are computed by

$$\pi_g = \left(\frac{p_g}{p_0}\right)^{R/C_p}, \qquad \text{(E1-2-2)}$$

$$\pi = \pi_g - \frac{gG^{\frac{1}{2}}}{C_p}\int_0^z \frac{1}{\theta_v} dz^*, \qquad \text{(E1-2-3)}$$

$$P(x,y,z^*) = (p_0 \pi^{C_p/R} - \overline{p})\, G^{\frac{1}{2}}, \qquad \text{(E1-2-4)}$$

where $p_0 = 1000$ hPa, θ_v is the interpolated virtual potential temperature of the outer model, and p_g is the pressure at the ground surface of the nested model, which is evaluated by

$$p_g = p_{go}\left\{1 - \frac{\Gamma(z_s - z_{so})}{T_{go}}\right\}^{\frac{g}{R\Gamma}}. \qquad \text{(E1-2-5)}$$

Here, p_{go} is the interpolated surface pressure of the outer model, and z_s and z_{so} denote the ground heights of the nested model and outer model, respectively. T_{go} is the ground temperature of the outer model. The ground temperature of the nested model is derived from the outer model but is adjusted according to the difference of the heights between the two models. We use 6.5×10^{-3} deg/m for Γ, the temperature lapse rate near the surface. When the pressure is determined, the density is calculated by

$$\rho(x,y,z^*) = \frac{p_0}{R\theta_m}\left(\frac{\overline{p} + P/G^{\frac{1}{2}}}{p_0}\right)^{C_v/C_p}. \qquad \text{(E1-2-6)}$$

Currently, the water quantities, other than water vapor, are regarded as zero in the initial field, so θ_v is used instead of the mass-virtual potential temperature in the denominator of (E1-2-6).

Using (C2-1-9) and (C2-1-10), the initial guess of the horizontal winds is calculated from the interpolated horizontal wind field of the outer model (u_0, v_0) as

$$U_0 = \frac{\rho G^{\frac{1}{2}}}{m} u_0,$$
$$V_0 = \frac{\rho G^{\frac{1}{2}}}{m} v_0, \qquad \text{(E1-2-7)}$$

In the initiation procedure, the above initial guess of the horizontal winds is modified as described in the next subsection.

P.G. The vertical profile of the reference atmosphere is set by sub.SETREF for the stand-alone case and by HRMEAN for the nesting case. The profile is interpolated into model planes by sub.CPTRFT.

E-2. Start-up procedure

E-2-1 Adjustment of the horizontal wind components

In anelastic models, mass conservation in the entire model domain becomes the solvability condition for the Poisson type pressure diagnostic equation, as Ikawa and Saito (1991) discussed. In a fully compressible model, the conservation of mass is not the solvability condition for the pressure equation, but total mass flux through lateral boundaries is still very important for maintaining the total mass in the model domain.

Volume integrating the continuity equation (C2-1-8), we obtain the following relation between the time tendency of total mass in the entire model domain and the mass flux through the lateral boundaries:

$$\frac{\partial}{\partial t}\iiint \frac{\rho G^{\frac{1}{2}}}{m^2} dxdydz^* = \left(\iint Udydz^*\right)_{x=0} - \left(\iint Udydz^*\right)_{x=X} + \left(\iint Vdxdz^*\right)_{y=0} - \left(\iint Vdxdz^*\right)_{y=Y} - \iint \frac{1}{m^2}(\rho_a V_r q_r + \rho_a V_s q_s + \rho_a V_g q_g)_{z^*=0} dxdy, \quad \text{(E2-1-1)}$$

where X and Y denote the dimensions of the nested model domain for x and y directions and we assume that W^* becomes zero at the lower and upper boundaries. The last term on right-hand side of above equation comes from the volume integration of PRC in (C2-1-8), which corresponds to the total surface precipitation in the model domain.[1]

In the nesting procedure, simple application of the interpolated wind of the mother model (E1-2-7) does not satisfy the above relation due to interpolation errors, differences of upper boundary conditions between the two models, and treatment of the PRC term in the nonhydrostatic model. In order to satisfy (E2-1-1), the interpolated winds U_0 and V_0 are adjusted by

$$U_0'(x,y,z^*) = U_0(x,y,z^*) + \frac{X-2x}{X}ADJ,$$
$$V_0'(x,y,z^*) = V_0(x,y,z^*) + \frac{Y-2y}{Y}ADJ, \quad \text{(E2-1-2)}$$

where ADJ is the difference between the expected change of total mass and the right-hand side of (E2-1-1):

$$ADJ = \frac{1}{2(S_{xz}+S_{yz})}\{S_{xy}\frac{\partial}{\partial t}\frac{\overline{p_s}-\overline{p_H}}{g} - \left(\iint U_0 dydz^*\right)_{x=0} + \left(\iint U_0 dydz^*\right)_{x=X}, - \left(\iint V_0 dxdz^*\right)_{y=0} + \left(\iint V_0 dxdz^*\right)_{y=Y} + \iint \frac{1}{m^2}(\rho_a V_r q_r + \rho_a V_s q_s + \rho_a V_g q_g)_{z^*=0} dxdy\}. \quad \text{(E2-1-3)}$$

Here, S_{xz} and S_{yz} are the square measures of the lateral planes of the nested model at the north-south and the east-west boundaries respectively, and S_{xy} is the area of the model domain. p_s and p_H are the averaged pressures

[1] Saito (1997) neglected this term. However, this term is not necessarily negligible in case of heavy precipitation or long-term simulation since mean precipitation of 10 mm corresponds to a pressure decrease of 1 hPa. For the GCSS WG4 Case-1 squall line simulation (Saito and Yamasaki, 1997; Redelsperger *et al.*, 2000), neglect of this term resulted in about a 2 hPa deficit of the mean pressure (Saito, 1998). More strictly, we should consider the surface vapor fluxes too, but currently neglect them.

of the outer model at the surface and top of the nested model. This adjustment is applied not only at the initial time of the nested model but also at all output times of the outer model.

E-2-2 Vertical wind component

The first guess of the vertical wind is calculated from the interpolated horizontal wind field of the outer model (U_0, V_0) using the continuity equation. In order to satisfy the boundary conditions of $W^*=0$ at the upper and lower boundaries, the following weighted mean vertical velocity ω_0 is introduced:

$$W^*_0(z^*) = W^*_u\frac{H-z^*}{H} + W^*_d\frac{z^*}{H}, \qquad \text{(E2-2-1)}$$

where W^*_u and W^*_d are the vertical velocities obtained by the upward and downward integration of the continuity equation:

$$W^*_u(z^*) = -m\int_0^{z^*}\left(\frac{\partial U_0'}{\partial x}+\frac{\partial V_0'}{\partial y}\right)dz_0^*, \qquad \text{(E2-2-2)}$$

$$W^*_d(z^*) = m\int_{z^*}^{H}\left(\frac{\partial U_0'}{\partial x}+\frac{\partial V_0'}{\partial y}\right)dz_0^*. \qquad \text{(E2-2-3)}$$

W^*_0 given by (E2-2-1) is mainly derived by upward integration near the surface and by downward integration near the upper boundary.

E-2-3 Mass-consistent variational calculus

The *hybrid* vertical wind W^*_0 given by (E2-2-1) does not necessarily satisfy the continuity equation. In order to obtain a three-dimensional, mass-consistent wind field, the variational calculus method (Saito, 1994) is available as an optional choice in the start-up procedure of the anelastic system.

The function needed to minimize the variance of the difference between the adjusted and the interpolated wind is described by the following equation (Sherman, 1978):

$$J=\int_V\{\alpha_1^2(U-U_0')+\alpha_1^2(V-V_0')^2+\alpha_2^2(W^*-W^*_0)^2$$
$$+\lambda DIVT(U,V,W)\}dxdydz^*. \qquad \text{(E2-3-1)}$$

Here, α_1 and α_2 are weight parameters that depend on the accuracy of the initial field, and λ is the Lagrange multiplier and is a function of x, y, and z^*. The variation of J is

$$\delta J=\int_V\{\alpha_1^2 2(U-U_0')\delta U+\alpha_1^2 2(V-V_0')\delta V+\alpha_2^2 2(W^*-W^*_0)\delta W^*+\lambda\left(\frac{\partial\delta U}{\partial x}+\frac{\partial\delta V}{\partial y}\right.$$
$$\left.+\frac{\partial\delta W^*}{\partial z^*}\right)\}dxdydz^*. \qquad \text{(E2-3-2)}$$

Note that the map factor is neglected in the divergence of the anelastic system (see C-2-3). Taking partial integration of (E2-3-2) and assuming boundary conditions

$$(\lambda\delta U)|_{0,x}=0,\ \ (\lambda\delta V)|_{0,Y}=0,\ \ (\lambda\delta W^*)|_{0,H}=0, \qquad \text{(E2-3-3)}$$

we obtain the following associated Euler-Lagrange equations whose solution minimizes (E2-3-1):

$$2\alpha_1^2(U-U_0')-\frac{\partial\lambda}{\partial x}=0, \qquad \text{(E2-3-4)}$$

$$2\alpha_1^2(V-V_0')-\frac{\partial\lambda}{\partial y}=0, \qquad \text{(E2-3-5)}$$

$$2\alpha_2{}^2(W^* - W^*{}_0) - \frac{\partial \lambda}{\partial z^*} = 0. \quad \text{(E2-3-6)}$$

Substituting the above relations into the constraint (C2-3-1), the equation of λ is obtained by solving the following Poisson equation :

$$\frac{\partial^2 \lambda}{\partial x^2} + \frac{\partial^2 \lambda}{\partial y^2} + \frac{\alpha_1{}^2}{\alpha_2{}^2}\frac{\partial^2 \lambda}{\partial z^{*2}} = -2\alpha_1{}^2 DIVT(U_0', V_0', W_0'), \quad \text{(E2-3-7)}$$

The boundary conditions of (E2-3-7) are $\lambda = 0$ at the lateral boundaries, and $\partial\lambda / \partial z^* = 0$ at the upper and lower boundaries. In the model, $\alpha_1 = 1$ and $\alpha^2 = \alpha_1{}^2/\alpha_2{}^2$ are usually used, and the solution of (E2-3-7) is obtained by the Successive Over-Relaxation method.

E-2-4 Initialization of pressure in elastic models

The variational calculus presented in the former section is applied only to anelastic systems. For an elastic model, the divergence in the initial field causes sound waves that are soon reduced and do not affect the later simulation results in most cases. The initial pressure field of the compressible model is given by (E1-2-4). For another way to initiate the pressure field, the model has an option in which the pressure field is given by solving the Poisson type pressure diagnostic equation for an anelastic system. This option efficiently smoothes start-up and minimizes the sound waves ; it is suitable for cases in which the surface pressure field of the outer model is not reliable. Currently, this option is not complete for initialization of the regional prediction model since the map factor is not considered in the anelastic pressure equation.

P.G. The wind field is adjusted according to the total mass tendency in sub.ADJFLX. Variational calculus is performed in sub.RELAX and sub.CADJP1.