2. Description of the Model

2. 1 Governing equations

The model is based on the primitive equations formulated in spherical coordinates λ, ϕ, and z, where λ is longitude, ϕ latitude, and z height. The vertical coordinate z is positive upward, with the ocean surface $z=0$. The hydrostatic and Boussinesq approximations are used. Hence, the variation of density is neglected in the momentum equations everywhere except in the buoyancy force. The subgrid-scale processes are parameterized by down-gradient mixing hypothesis, where the exchange coefficients are assumed to be constant. Let u, v, and w be the zonal, meridional, and vertical velocity components, respectively. The equations of horizontal motion are

$$\frac{\partial u}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial u}{\partial \lambda}+\frac{v}{a}\frac{\partial u}{\partial \phi}+w\frac{\partial u}{\partial z}-\frac{uv\tan\phi}{a}-2\Omega v\sin\phi$$

$$=\frac{-1}{\rho_0 a\cos\phi}\frac{\partial p}{\partial \lambda}+A_m\left\{\nabla^2 u+\frac{(1-\tan^2\phi)}{a^2}u-\frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial \lambda}\right\}+K_m\frac{\partial^2 u}{\partial z^2}, \tag{2-1}$$

$$\frac{\partial v}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial v}{\partial \lambda}+\frac{v}{a}\frac{\partial v}{\partial \phi}+w\frac{\partial v}{\partial z}+\frac{u^2\tan\phi}{a}+2\Omega u\sin\phi$$

$$=\frac{-1}{\rho_0 a}\frac{\partial p}{\partial \phi}+A_m\left\{\nabla^2 v+\frac{(1-\tan^2\phi)}{a^2}v+\frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial \lambda}\right\}+K_m\frac{\partial^2 v}{\partial z^2}, \tag{2-2}$$

and the hydrostatic equation is

$$\frac{\partial p}{\partial z}=-\rho g, \tag{2-3}$$

where t is time, a the earth' s radius, Ω the angular velocity of the earth' s rotation, ρ_0 a constant reference density, p the pressure, A_m the coefficient of horizontal eddy viscosity, K_m the coefficient of vertical eddy viscosity, ρ the density, g the acceleration of gravity, and ∇^2 the horizontal Laplacian operator,

$$\nabla^2=\frac{1}{a^2\cos^2\phi}\frac{\partial^2}{\partial \lambda^2}+\frac{1}{a^2\cos\phi}\frac{\partial}{\partial \phi}\left(\cos\phi\frac{\partial}{\partial \phi}\right). \tag{2-4}$$

The equation of continuity is

$$\frac{1}{a\cos\phi}\frac{\partial u}{\partial \lambda}+\frac{1}{a\cos\phi}\frac{\partial}{\partial \phi}(v\cos\phi)+\frac{\partial w}{\partial z}=0. \tag{2-5}$$

The equations for the conservation of heat and salt are

$$\frac{\partial T}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial T}{\partial \lambda}+\frac{v}{a}\frac{\partial T}{\partial \phi}+w\frac{\partial T}{\partial z}=A_h\nabla^2 T+\frac{K_h}{\delta}\frac{\partial^2 T}{\partial z^2}, \tag{2-6}$$

$$\frac{\partial S}{\partial t}+\frac{u}{a\cos\phi}\frac{\partial S}{\partial \lambda}+\frac{v}{a}\frac{\partial S}{\partial \phi}+w\frac{\partial S}{\partial z}=A_h\nabla^2 S+\frac{K_h}{\delta}\frac{\partial^2 S}{\partial z^2}, \qquad (2\text{-}7)$$

where T is the temperature, S the salinity, A_h the coefficient of horizontal eddy diffusivity, K_h the coefficient of vertical eddy diffusivity, and the coefficient δ is defined as

$$\delta\equiv \begin{matrix}0\\1\end{matrix} \quad \text{for} \quad \frac{\partial\rho}{\partial z}\gtreqless 0\,. \qquad (2\text{-}8)$$

δ is introduced for the convective overturning when the stratification is unstable.

As the equation of state, Eckart's approximation formula (Bryan, 1969b) is used. If ρ, ρ_0, g, and z are given in cgs units, T in degrees Celsius, and S in parts per thousand, the formula reads

$$\rho=\frac{P'+P_0}{1.000027[A+0.698(P'+P_0)]}, \qquad (2\text{-}9)$$

where P', P_0, and A are defined as follows :

$$P'=\frac{\rho_0 g|z|}{1.013\times10^6}+1.0\,,$$

$$P_0=5890+38T-0.375T^2+3S\,, \qquad (2\text{-}10)$$

$$A=1779.5+11.25T-0.0745T^2-(3.8+0.01T)S\,.$$

2. 2 Model domain and boundary conditions

The model ocean is bounded by two meridians, 100° of longitude apart, and extends from 30° S to 54° N. It has a flat bottom of 5km depth. This domain is considered as the size of the Pacific Ocean. Fig. 2-1 schematically represents it.

The boundary conditions on the velocity are zero normal velocity and no slip at the western and eastern walls, and zero normal velocity and free slip at the southern and northern walls, i. e.,

$$u=v=0 \qquad \text{at } \lambda=0^\circ,\ 100^\circ, \qquad (2\text{-}11)$$

$$\frac{\partial u}{a\partial\phi}=v=0 \qquad \text{at } \phi=-30^\circ,\ 54^\circ. \qquad (2\text{-}12)$$

There is neither heat flux nor salinity flux through the lateral walls, i. e.,

$$\frac{\partial T}{a\cos\phi\partial\lambda}=\frac{\partial S}{a\cos\phi\partial\lambda}=0 \qquad \text{at } \lambda=0^\circ,\ 100^\circ, \qquad (2\text{-}13)$$

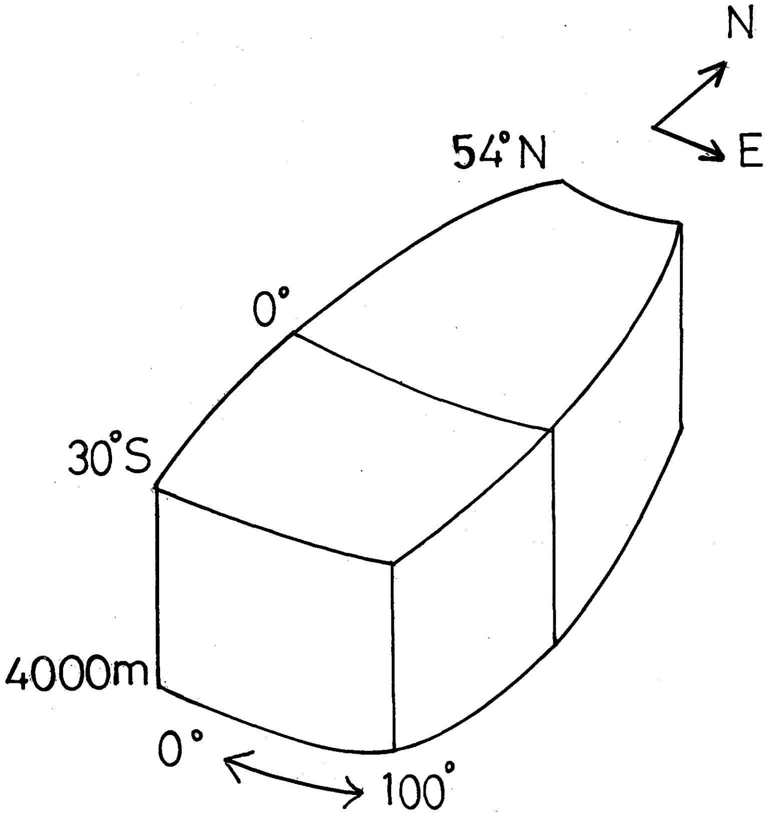

Fig. 2-1 The model domain.

$$\frac{\partial T}{a\partial\phi}=\frac{\partial S}{a\partial\phi}=0 \qquad \text{at } \phi=-30^\circ,\ 54^\circ . \qquad (2\text{-}14)$$

At the ocean bottom, the vertical velocity, heat flux and salinity flux are taken to be zero ; thus

$$\left.\begin{aligned} &w=0\ , \\ &K_h\frac{\partial T}{\partial z}=K_h\frac{\partial S}{\partial z}=0\ , \\ &K_m\frac{\partial u}{\partial z}=\tau_B^\lambda/\rho_0\ , \\ &K_m\frac{\partial v}{\partial z}=\tau_B^\phi/\rho_0\ . \end{aligned}\right\} \quad \text{at } z=-H . \qquad (2\text{-}15)$$

The bottom stress vector $(\tau_B^\lambda,\ \tau_B^\phi)$ is determined on the assumption that the bottom Ekman layer is embedded in the lowest layer of the model ocean in such a way as to satisfy the no-slip condition at the bottom, i. e.,

$$\tau_B^\lambda = \sqrt{\Omega \sin\phi\, K_m}\,(u_B - v_B)$$

$$(\phi \geqq 0),$$

$$\tau_B^\phi = \sqrt{\Omega \sin\phi\, K_m}\,(u_B + v_B)$$

or (2-16)

$$\tau_B^\lambda = \sqrt{-\Omega \sin\phi\, K_m}\,(u_B + v_B)$$

$$(\phi < 0),$$

$$\tau_B^\phi = \sqrt{-\Omega \sin\phi\, K_m}\,(-u_B + v_B)$$

where u_B and v_B are the horizontal components of the velocity at the lowest layer or at the top of the Ekman layer.

At the ocean surface, the rigid-lid approximation is made to filter out external gravity waves, and the wind stress $(\tau^\lambda, \tau^\phi)$, heat flux (Q_T), and salinity flux (Q_S) are specified, i. e.,

$$
\begin{aligned}
&w = 0\ ,\\
&K_m \frac{\partial u}{\partial z} = \tau^\lambda/\rho_0\ ,\\
&K_m \frac{\partial v}{\partial z} = \tau^\phi/\rho_0\ , \qquad \text{at } z=0\ , \qquad (2\text{-}17)\\
&K_h \frac{\partial T}{\partial z} = Q_T/\rho_0 c\ ,\\
&K_h \frac{\partial S}{\partial z} = Q_S\ ,
\end{aligned}
$$

where c is the specific heat of the sea water. τ^λ, τ^ϕ, Q_T, and Q_S will be described in section 2.5.

2.3 Prognostic equations

Using the hydrostatic relation (2-3), the pressure at any depth z is given by

$$p = p_s + \int_z^0 g\rho\, dz\ , \tag{2-18}$$

where p_s is the pressure at the ocean surface. In terms of p_s, Eqs. (2-1) and (2-2) can be rewritten as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho_0 a\cos\phi}\frac{\partial p_s}{\partial \lambda} + U + 2\Omega v \sin\phi\ , \tag{2-19}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho_0 a}\frac{\partial p_s}{\partial \phi} + V - 2\Omega u \sin\phi\ , \tag{2-20}$$

where

$$U=-\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial\lambda}\int_z^0 g\rho dz-\frac{u}{a\cos\phi}\frac{\partial u}{\partial\lambda}-\frac{v}{a}\frac{\partial u}{\partial\phi}-w\frac{\partial u}{\partial z}+\frac{uv\tan\phi}{a}$$

$$+A_m\left\{\nabla^2 u+\frac{(1-\tan^2\phi)}{a^2}u-\frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial v}{\partial\lambda}\right\}+K_m\frac{\partial^2 u}{\partial z^2}, \qquad (2\text{-}21)$$

$$V=-\frac{1}{\rho_0 a}\frac{\partial}{\partial\phi}\int_z^0 g\rho dz-\frac{u}{a\cos\phi}\frac{\partial v}{\partial\lambda}-\frac{v}{a}\frac{\partial v}{\partial\phi}-w\frac{\partial v}{\partial z}-\frac{u^2\tan\phi}{a}$$

$$+A_m\left\{\nabla^2 v+\frac{(1-\tan^2\phi)}{a^2}v+\frac{2\sin\phi}{a^2\cos^2\phi}\frac{\partial u}{\partial\lambda}\right\}+K_m\frac{\partial^2 v}{\partial z^2}, \qquad (2\text{-}22)$$

To integrate the above equations without p_S equation, the horizontal motion (u, v) is decomposed according to Bryan (1969a) as

$$u=\bar{u}+u', \qquad (2\text{-}23)$$

$$v=\bar{v}+v', \qquad (2\text{-}24)$$

where $\bar{u}$ and $\bar{v}$ are the vertically averaged velocity components, and u' and v' the deviations from them. By the horizontal nondivergence of the vertical mean current $(\bar{u}, \bar{v})$, stream function Ψ can be defined such that

$$\bar{u}=\frac{1}{H}\int_{-H}^0 u dz=-\frac{1}{Ha}\frac{\partial\Psi}{\partial\phi}, \qquad (2\text{-}25)$$

$$\bar{v}=\frac{1}{H}\int_{-H}^0 v dz=\frac{1}{Ha\cos\phi}\frac{\partial\Psi}{\partial\lambda}. \qquad (2\text{-}26)$$

A prediction equation for Ψ is obtained then by taking the vertical averages of Eqs. (2-19) and (2-20) and eliminating p_S. The result is

$$\frac{1}{a^2\cos^2\phi}\frac{\partial}{\partial\lambda}\left(\frac{1}{H}\frac{\partial^2\Psi}{\partial\lambda\partial t}\right)+\frac{1}{a^2\cos\phi}\frac{\partial}{\partial\phi}\left(\frac{\cos\phi}{H}\frac{\partial^2\Psi}{\partial\phi\partial t}\right)$$

$$=\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{H}\int_{-H}^0(V-2\Omega u\sin\phi)dz\right\}$$

$$-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left\{\frac{\cos\phi}{H}\int_{-H}^0(U+2\Omega v\sin\phi)dz\right\}. \qquad (2\text{-}27)$$

When H is constant, this equation becomes the Poisson equation :

$$\nabla^2\frac{1}{H}\frac{\partial\Psi}{\partial t}=\frac{1}{a\cos\phi}\frac{\partial}{\partial\lambda}\left\{\frac{1}{H}\int_{-H}^0(V-2\Omega u\sin\phi)dz\right\}$$

$$-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left\{\frac{\cos\phi}{H}\int_{-H}^{0}(U+2\Omega v\sin\phi)dz\right\}. \tag{2-28}$$

From Eqs. (2-19) and (2-20) prediction equations for the vertical shear current are obtained:

$$\frac{\partial u'}{\partial t}=U-\frac{1}{H}\int_{-H}^{0}Udz+2\Omega v'\sin\phi\ , \tag{2-29}$$

$$\frac{\partial v'}{\partial t}=V-\frac{1}{H}\int_{-H}^{0}Vdz-2\Omega u'\sin\phi\ . \tag{2-30}$$

The temperature and salinity are predicted from Eqs. (2-6) and (2-7), namely,

$$\frac{\partial T}{\partial t}=-\frac{u}{a\cos\phi}\frac{\partial T}{\partial\lambda}-\frac{v}{a}\frac{\partial T}{\partial\phi}-w\frac{\partial T}{\partial z}+A_h\nabla^2T+\frac{K_h}{\delta}\frac{\partial^2T}{\partial z^2}, \tag{2-31}$$

$$\frac{\partial S}{\partial t}=-\frac{u}{a\cos\phi}\frac{\partial S}{\partial\lambda}-\frac{v}{a}\frac{\partial S}{\partial\phi}-w\frac{\partial S}{\partial z}+A_h\nabla^2S+\frac{K_h}{\delta}\frac{\partial^2S}{\partial z^2}. \tag{2-32}$$

2.4 Grid system and finite difference equations

The finite difference methods used to solve the equations follow those of Semtner (1974), and partly Han (1975). In the following section, the subscripts i, j and k always represent the longitudinal, latitudinal and vertical indices, respectively.

2.4.1 Grid system

The ocean is composed of KM layers, and the grid points are irregularly spaced in the vertical direction. The horizontal spacing of the grid points corresponds to the increments $\Delta\lambda$ and $\Delta\phi$ in the longitudinal and latitudinal directions, respectively.

The arrangement of the variables in the vertical direction is shown in Fig. 2-2. The horizontal velocity components u and v, temperature T, salinity S, density ρ and pressure p are located at the levels denoted by z_k (<0) $(k=1, \cdots, KM)$.

The intervals between the levels are defined as

$$\begin{aligned}&\Delta z_{1/2}=-z_1\ ,\\&\Delta z_{k-1/2}=z_{k-1}-z_k \qquad (k=2,\cdots,KM),\\&\Delta z_{KM+1/2}=z_{KM}+H\ .\end{aligned} \tag{2-33}$$

The thickness of the layers is defined as

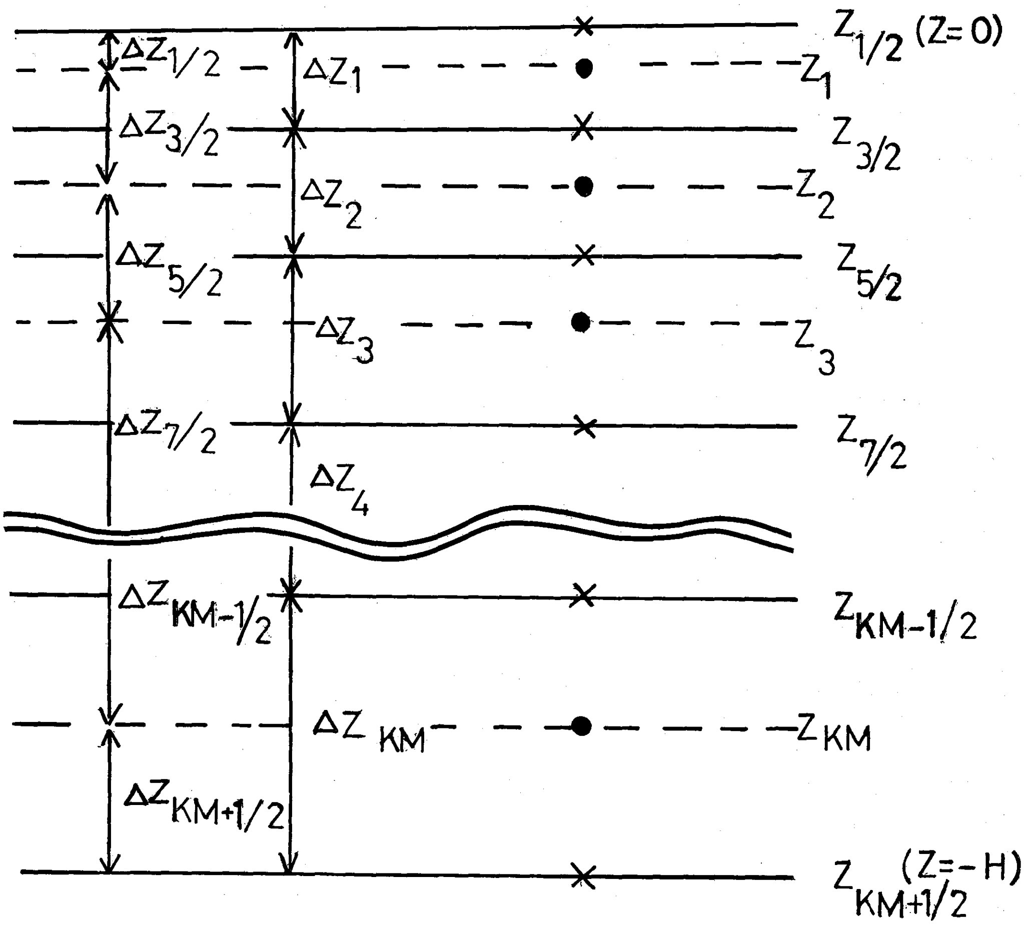

Fig. 2-2 Vertical placement of the variables. Dots are for u, v, T, S, ρ, and p; crosses are for w.

$$
\begin{aligned}
&\Delta z_1 = \Delta z_{1/2} + 1/2\,\Delta z_{3/2}\,, \\
&\Delta z_k = 1/2(\Delta z_{k-1/2} + \Delta z_{k+1/2}) \qquad (k=2, \cdots, KM-1)\,, \\
&\Delta z_{KM} = 1/2\,\Delta z_{KM-1/2} + \Delta z_{KM+1/2}\,.
\end{aligned}
\tag{2-34}
$$

The vertical velocity w is carried at the intermediate levels

$$z_{k+1/2}(k=0, \cdots, KM)\,,$$

where

$$
\begin{aligned}
&z_{1/2} = 0\,, \\
&z_{k+1/2} = 1/2(z_k + z_{k+1}) \qquad (k=1, \cdots, KM-1)\,,
\end{aligned}
\tag{2-35}
$$

$z_{KM+1/2} = -H$.

A staggered grid system in the horizontal plane is shown in Fig. 2-3. The temperature T, salinity S, density ρ, pressure p, and stream function Ψ are located at integer grid points (i, j) $(i=1, \cdots, IM, j=1, \cdots JM)$, where IM and JM are the total numbers of grid points in the longitudinal and latitudinal directions, respectively. The horizontal velocitly components u and v are located at half-integer grid points $(i+1/2, j+1/2)$ $(i=1, \cdots, IM-1, j=1, \cdots, JM-1)$. The vertical velocity in the prognostic equations for T and S is evaluated at (i, j), while that for u and v is evaluated at $(i+1/2, j+1/2)$. The coastlines of the model ocean are placed on (i, j) points, that is, $(2, j)$, $(IM-1, j)$ $(j=2, \cdots, JM-1)$, and $(i, 2)$, $(i, JM-1)$ $(i=2, \cdots, IM-1)$. The grid points outside the coastlines are only used for specifying boundary conditions. The horizontal grid distances Δx_j, $\Delta x_{j+1/2}$, and Δy are defined as

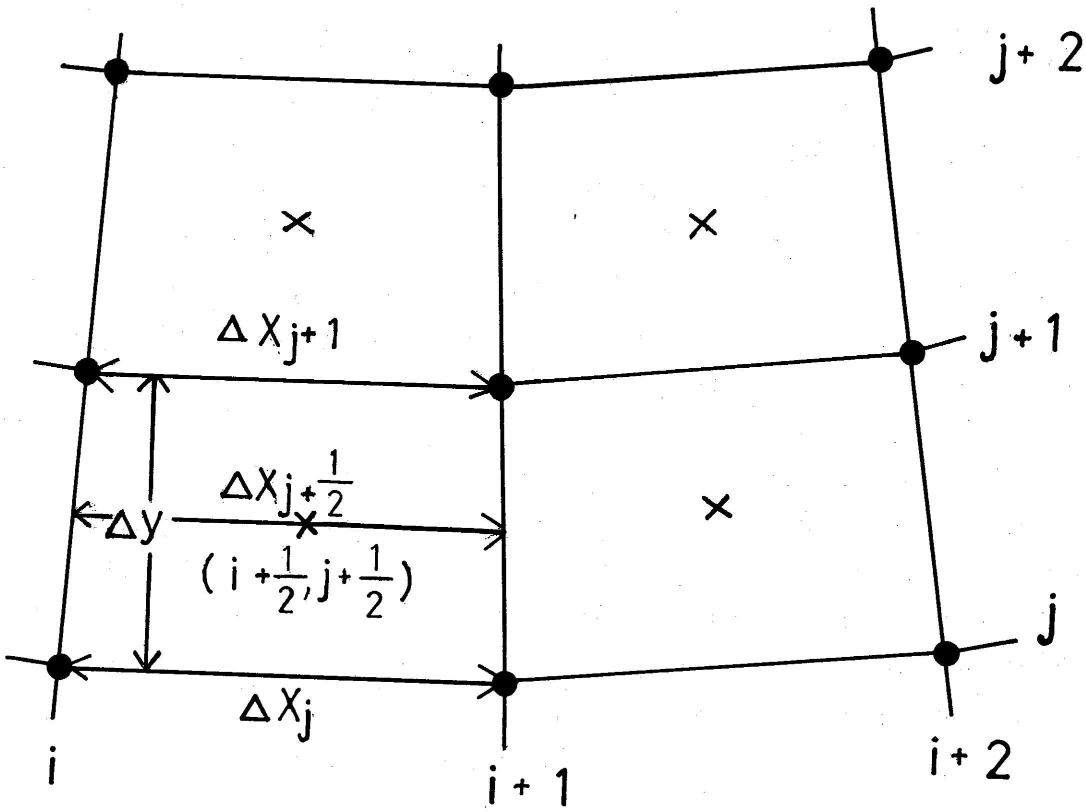

Fig. 2-3 Horizontal placement of the variables. Dots are for T, S, ρ, p, Ψ, and w in the T and S equations; crosses are for u, v, and w in the u and v equations.

$$\Delta x_j = a\cos\phi_j \Delta\lambda ,$$
$$\Delta x_{j+1/2} = 1/2(\Delta x_j + \Delta x_{j+1}) , \tag{2-36}$$
$$\Delta y = a\Delta\phi .$$

where ϕ_j is the latitude of j point. The trigonometric functions at $j+1/2$ point are given using $\cos\phi_j$ at j point by

$$\cos\phi_{j+1/2} = 1/2(\cos\phi_j + \cos\phi_{j+1}) ,$$
$$\sin\phi_{j+1/2} = (\cos\phi_j - \cos\phi_{j+1})/\Delta y , \tag{2-37}$$
$$\tan\phi_{j+1/2} = \sin\phi_{j+1/2}/\cos\phi_{j+1/2} .$$

The area elements Π_j and $\Pi_{j+1/2}$ are defined as

$$\Pi_j = \Delta x_j \Delta y ,$$
$$\Pi_{j+1/2} = \Delta x_{j+1/2} \Delta y . \tag{2-38}$$

2.4.2 Time differencing

The principal time differencing utilized in the model is the leapfrog scheme. Eyery ten time steps, however, a forward scheme is applied to suppress the time splitting associated with the leapfrog scheme. The friction and diffusion terms are always evaluated with a forward scheme. In addition, a trapezoidal implicit scheme is used for the Coriolis terms in order to render inertial oscillations stable with a long time step.

2.4.3 Momentum equations

The finite difference analogs of the equations of motion for the vertical shear current are obtained by rendering Eqs. (2-29) and (2-30) in the finite difference form. In this subsection, notation α is used to denote either u or v.

(a) Pressure gradient

The pressure gradient forces at the kth level relative to those at the first level are written as

$$(P\lambda)_{i+1/2,j+1/2,1} = 0 ,$$
$$(P\lambda)_{i+1/2,j+1/2,k} = -\frac{1}{2}\frac{1}{\rho_0}\sum_{l=1}^{k-1}(\rho_{i+1,j+1,l+1/2} + \rho_{i+1,j,l+1/2} - \rho_{i,j+1,l+1/2} - \rho_{i,j,l+1/2}) \tag{2-39}$$

$$\cdot g\Delta z_{l+1/2}/\Delta x_{j+1/2} \qquad (k=2, \cdots, KM),$$

and

$$(P\phi)_{i+1/2,j+1/2,1}=0,$$

$$(P\phi)_{i+1/2,j+1/2,k}=-\frac{1}{2}\frac{1}{\rho_0}\sum_{l=1}^{k-1}(\rho_{i+1,j+1,l+1/2}+\rho_{i,j+1,l+1/2}-\rho_{i+1,j,l+1/2}-\rho_{i,j,l+1/2}) \qquad (2\text{-}40)$$

$$\cdot g\Delta z_{l+1/2}/\Delta y \qquad (k=2, \cdots, KM),$$

where $\rho_{i,j,k+1/2}$ is defined as

$$\rho_{i,j,k+1/2}=\frac{1}{2}(\rho_{i,j,k}+\rho_{i,j,k+1}), \qquad (2\text{-}41)$$

and the density $\rho_{i,j,k}$ is calculated from the equation of state (2-9) with $T_{i,j,k}$, $S_{i,j,k}$ and z_k.

(b) Horizontal advection

It is convenient to define the following volume fluxes per unit depth in advance (Fig. 2-4 (a)):

$$(FUC)_{i,j,k}=\frac{1}{4}\Delta y(u_{i-1/2,j-1/2,k}+u_{i+1/2,j-1/2,k}+u_{i-1/2,j+1/2,k}+u_{i+1/2,j+1/2,k}),$$

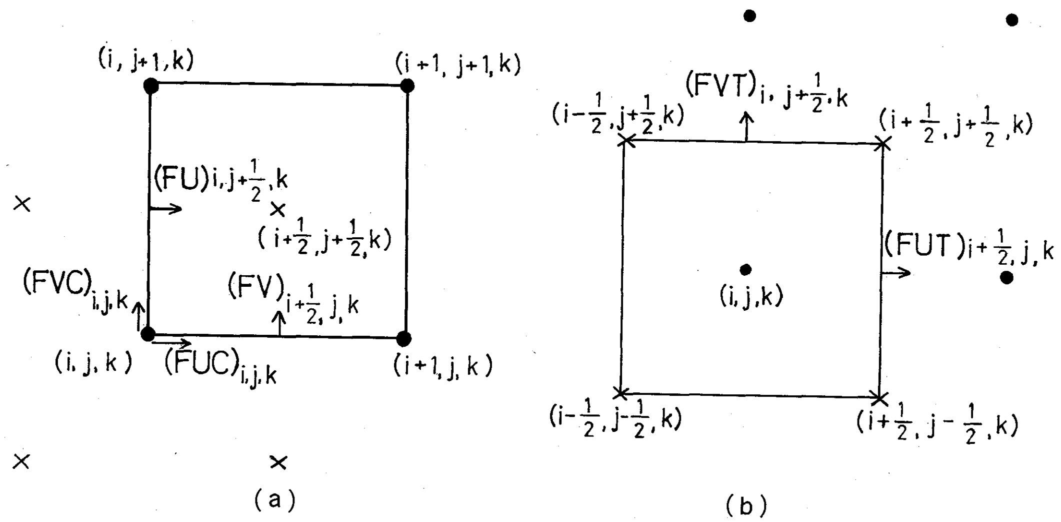

Fig. 2-4 Location of the horizontal volume fluxes defined to obtain finite-difference forms of the horizontal advection terms in (a) the momentum equations and (b) temperature and salinity equations.

$$(FVC)_{i,j,k}=\frac{1}{4}\{\Delta x_{j-1/2}(v_{i-1/2,j-1/2,k}+v_{i+1/2,j-1/2,k})$$
$$+\Delta x_{j+1/2}(v_{i-1/2,j+1/2,k}+v_{i+1/2,j+1/2,k})\}\,,$$

(2-42)

$$(FU)_{i,j+1/2,k}=\frac{1}{2}\{(FUC)_{i,j,k}+(FUC)_{i,j+1,k}\}\,,$$

$$(FV)_{i+1/2,j,k}=\frac{1}{2}\{(FVC)_{i,j,k}+(FVC)_{i+1,j,k}\}\,.$$

Then the horizontal advection terms are given by

$$(M\lambda)_{i+1/2,j+1/2,k}=-\frac{1}{\Pi_{j+1/2}}\Big[\frac{2}{3}\{(uu)_{i+1,j+1/2,k}-(uu)_{i,j+1/2,k}$$
$$+(uv)_{i+1/2,j+1,k}-(uv)_{i+1/2,j,k}\}$$
$$+\frac{1}{3}\{(uc)_{i+1,j+1,k}-(uc)_{i,j,k}$$
$$+(uc)_{i,j+1,k}-(uc)_{i+1,j,k}\}\Big]\,,$$

(2-43)

$$(M\phi)_{i+1/2,j+1/2,k}=-\frac{1}{\Pi_{j+1/2}}\Big[\frac{2}{3}\{(vu)_{i+1,j+1/2,k}-(vu)_{i,j+1/2,k}$$
$$+(vv)_{i+1/2,j+1,k}-(vv)_{i+1/2,j,k}\}$$
$$+\frac{1}{3}\{(vc)_{i+1,j+1,k}-(vc)_{i,j,k}$$
$$+(vc)_{i,j+1,k}-(vc)_{i+1,j,k}\}\Big]\,,$$

where

$$(\alpha u)_{i,j+1/2,k}=\frac{1}{2}(\alpha_{i-1/2,j+1/2,k}+\alpha_{i+1/2,j+1/2,k})\,(FU)_{i,j+1/2,k}\,,$$

$$(\alpha v)_{i+1/2,j,k}=\frac{1}{2}(\alpha_{i+1/2,j-1/2,k}+\alpha_{i+1/2,j+1/2,k})\,(FV)_{i+1/2,j,k}\,,$$

$$(\alpha c)_{i+1,j+1,k}=\frac{1}{4}(\alpha_{i+1/2,j+1/2,k}+\alpha_{i+3/2,j+3/2,k})\{(FUC)_{i+1,j+1,k}+(FVC)_{i+1,j+1,k}\}\,,$$

(2-44)

$$(\alpha c)_{i,j,k}=\frac{1}{4}(\alpha_{i-1/2,j-1/2,k}+\alpha_{i+1/2,j+1/2,k})\{(FUC)_{i,j,k}+(FVC)_{i,j,k}\}\,,$$

$$(\alpha c)_{i,j+1,k}=\frac{1}{4}(\alpha_{i+1/2,j+1/2,k}+\alpha_{i-1/2,j+3/2,k})\{-(FUC)_{i,j+1,k}+(FVC)_{i,j+1,k}\}\,,$$

$$(\alpha c)_{i+1,j,k}=\frac{1}{4}(\alpha_{i+1/2,j+1/2,k}+\alpha_{i+3/2,j-1/2,k})\{-(FUC)_{i+1,j,k}+(FVC)_{i+1,j,k}\}\,.$$

(c) Vertical advection

Using the equation of continuity (2-5) and the upper boundary condition (2-17), the vertical velocity at $(i+1/2,\ j+1/2,\ k+1/2)$ is given by

$$w_{i+1/2,j+1/2,1/2}=0\,,$$

$$w_{i+1/2,j+1/2,k+1/2}=\frac{1}{\Pi_{j+1/2}}\sum_{l=1}^{k}\{(FU)_{i+1,j+1/2,l}-(FU)_{i,j+1/2,l}+(FV)_{i+1/2,j+1,l}-(FV)_{i+1/2,j,l}\}\Delta z_l \qquad (k=1,\cdots,KM)\,. \tag{2-45}$$

Then the vertical advection terms are written as

$$(W\lambda)_{i+1/2,j+1/2,k}=-\{(uw)_{i+1/2,j+1/2,k-1/2}-(uw)_{i+1/2,j+1/2,k+1/2}\}/\Delta z_k\,,$$
$$(W\phi)_{i+1/2,j+1/2,k}=-\{(vw)_{i+1/2,j+1/2,k-1/2}-(vw)_{i+1/2,j+1/2,k+1/2}\}/\Delta z_k\,, \tag{2-46}$$

where

$$(\alpha w)_{i+1/2,j+1/2,k+1/2}=\frac{1}{2}(\alpha_{i+1/2,j+1/2,k}+\alpha_{i+1/2,j+1/2,k+1})\cdot w_{i+1/2,j+1/2,k+1/2}\,. \tag{2-47}$$

(d) Horizontal friction

The finite difference analogs of the horizontal friction terms are

$$(F\lambda)_{i+1/2,j+1/2,k}=A_m\left\{(\nabla^2 u)_{i+1/2,j+1/2,k}+\frac{(1-\tan^2\phi_{j+1/2})}{a^2}u_{i+1/2,j+1/2,k}-\frac{\sin\phi_{j+1/2}}{a^2\cos^2\phi_{j+1/2}}\cdot\frac{v_{i+3/2,j+1/2,k}-v_{i-1/2,j+1/2,k}}{\Delta x_{j+1/2}}\right\},$$

$$(F\phi)_{i+1/2,j+1/2,k}=A_m\left\{(\nabla^2 v)_{i+1/2,j+1/2,k}+\frac{(1-\tan^2\phi_{j+1/2})}{a^2}v_{i+1/2,j+1/2,k}+\frac{\sin\phi_{j+1/2}}{a^2\cos^2\phi_{j+1/2}}\cdot\frac{u_{i+3/2,j+1/2,k}-u_{i-1/2,j+1/2,k}}{\Delta x_{j+1/2}}\right\}, \tag{2-48}$$

where

$$(\nabla^2\alpha)_{i+1/2,j+1/2,k}=\{(\alpha_{i+3/2,j+1/2,k}-\alpha_{i+1/2,j+1/2,k})-(\alpha_{i+1/2,j+1/2,k}-\alpha_{i-1/2,j+1/2,k})\}/\Delta x_{j+1/2}^2+\{\cos\phi_{j+1}(\alpha_{i+1/2,j+3/2,k}-\alpha_{i+1/2,j+1/2,k})-\cos\phi_j(\alpha_{i+1/2,j+1/2,k}-\alpha_{i+1/2,j-1/2,k})\}/\cos\phi_{j+1/2}\Delta y^2\,. \tag{2-49}$$

(e) Vertical friction

The vertical friction terms are written in the following finite difference form :

$$(G\lambda)_{i+1/2,j+1/2,k}=\{(Du)_{i+1/2,j+1/2,k-1/2}-(Du)_{i+1/2,j+1/2,k+1/2}\}/\Delta z_k\,,$$
$$(G\phi)_{i+1/2,j+1/2,k}=\{(Dv)_{i+1/2,j+1/2,k-1/2}-(Dv)_{i+1/2,j+1/2,k+1/2}\}/\Delta z_k\,, \tag{2-50}$$

where

$$(D\alpha)_{i+1/2,\,j+1/2,\,k+1/2}=K_m(\alpha_{i+1/2,\,j+1/2,\,k}-\alpha_{i+1/2,\,j+1/2,\,k})/\,\Delta z_{k+1/2}\,. \tag{2-51}$$

From the above, the finite difference analogs of Eqs. (2-29) and (2-30) for u' and v' at the time level $n+1$ are

$$\frac{u'^{(n+1)}_{i+1/2,\,j+1/2,\,k}-u'^{(n-1)}_{i+1/2,\,j+1/2,\,k}}{2\Delta t}=U^{(n,n-1)}_{i+1/2,\,j+1/2,\,k}-\frac{1}{H}\sum_{l=1}^{KM}U^{(n,n-1)}_{i+1/2,\,j+1/2,\,l}\,\Delta z_l$$

$$+2\Omega\sin\phi_{j+1/2}(\beta v'^{(n+1)}_{i+1/2,\,j+1/2,\,k}+\gamma v'^{(n-1)}_{i+1/2,\,j+1/2,\,k})\,, \tag{2-52}$$

$$\frac{v'^{(n-1)}_{i+1/2,\,j+1/2,\,k}-v'^{(n-1)}_{i+1/2,\,j+1/2,\,k}}{2\Delta t}=V^{(n,n-1)}_{i+1/2,\,j+1/2,\,k}-\frac{1}{H}\sum_{l=1}^{KM}V^{(n,n-1)}_{i+1/2,\,j+1/2,\,l}\,\Delta z_l$$

$$-2\Omega\sin\phi_{j+1/2}(\beta u'^{(n+1)}_{i+1/2,\,j+1/2,\,k}+\gamma u'^{(n-1)}_{i+1/2,\,j+1/2,\,k})\,, \tag{2-53}$$

where

$$U^{(n,n-1)}_{i+1/2,\,j+1/2,\,k}=(P\lambda)^{(n)}_{i+1/2,\,j+1/2,\,k}+(M\lambda)^{(n)}_{i+1/2,\,j+1/2,\,k}+(W\lambda)^{(n)}_{i+1/2,\,j+1/2,\,k}$$

$$+\frac{\tan\phi_{j+1/2}}{a}\,u^{(n)}_{i+1/2,\,j+1/2,\,k}\cdot v^{(n)}_{i+1/2,\,j+1/2,\,k}$$

$$+(F\lambda)^{(n-1)}_{i+1/2,\,j+1/2,\,k}+(G\lambda)^{(n-1)}_{i+1/2,\,j+1/2,\,k}\,, \tag{2-54}$$

$$V^{(n,n-1)}_{i+1/2,\,j+1/2,\,k}=(P\phi)^{(n)}_{i+1/2,\,j+1/2,\,k}+(M\phi)^{(n)}_{i+1/2,\,j+1/2,\,k}+(W\phi)^{(n)}_{i+1/2,\,j+1/2,\,k}$$

$$-\frac{\tan\phi_{j+1/2}}{a}(u^{(n)}_{i+1/2,\,j+1/2,\,k})^2$$

$$+(F\phi)^{(n-1)}_{i+1/2,\,j+1/2,\,k}+(G\phi)^{(n-1)}_{i+1/2,\,j+1/2,\,k}\,, \tag{2-55}$$

and superscript n indicates the time level n of the variable, and Δt denotes a time increment. Eqs. (2-52) and (2-53) are solved simultaneously for $u'^{(n+1)}_{i+1/2,\,j+1/2,\,k}$ and $v'^{(n+1)}_{i+1/2,\,j+1/2,\,k}$. The coefficients β and γ are taken to be 0.5 so that inertial oscillations are computationally neutral.

When the forward time difference is applied, the variables at time level n-1 in Eqs. (2-52) and (2-53) are replaced by those at time level n, and the denominator 2Δt on the lefthand side is replaced by Δt.

2.4.4 Temperature and salinity equations

As the temperature equation (2-31) and salinity equation (2-32) are similar to each other, only the finite difference analog of the temperature equation is written here.

(a) Horizontal advection

It is convenient to define the following volume fluxes per unit depth (see Fig. 2-4 (b)):

$$(FUT)_{i+1/2,j,k}=\frac{1}{2}\Delta y(u_{i+1/2,j-1/2,k}+u_{i+1/2,j+1/2,k}),$$

$$(FVT)_{i,j+1/2,k}=\frac{1}{2}\Delta x_{j+1/2}(v_{i-1/2,j+1/2,k}+v_{i+1/2,j+1/2,k}). \tag{2-56}$$

Then the horizontal advection terms are given as

$$(M_T)_{i,j,k}=-\frac{1}{\Pi_j}\{(Tu)_{i+1/2,j,k}-(Tu)_{i-1/2,j,k}+(Tv)_{i,j+1/2,k}-(Tv)_{i,j-1/2,k}\}, \tag{2-57}$$

where

$$(Tu)_{i+1/2,j,k}=\frac{1}{2}(T_{i,j,k}+T_{i+1,j,k})(FUT)_{i+1/2,j,k},$$

$$(Tv)_{i,j+1/2,k}=\frac{1}{2}(T_{i,j,k}+T_{i,j+1,k})(FVT)_{i,j+1/2,k}. \tag{2-58}$$

(b) Vertical advection

The vertical velocity at $(i,\ j,\ k+1/2)$ is given by

$$w_{i,j,1/2}=0,$$

$$w_{i,j,k+1/2}=\frac{1}{\Pi_j}\sum_{l=1}^{k}\{(FUT)_{i+1/2,j,l}-(FUT)_{i-1/2,j,l}+(FVT)_{i,j+1/2,l}-(FVT)_{i,j-1/2,l}\}\Delta z_l \quad (k=1,\cdots,KM). \tag{2-59}$$

Then the vertical advection term is written as

$$(W_T)_{i,j,k}=-\{(Tw)_{i,j,k-1/2}-(Tw)_{i,j,k+1/2}\}/\Delta z_k, \tag{2-60}$$

where

$$(Tw)_{i,j,k+1/2}=\frac{1}{2}(T_{i,j,k}+T_{i,j,k+1})\cdot w_{i,j,k+1/2}. \tag{2-61}$$

The vertical velocity $w_{i+1/2,j+1/2,k+1/2}$ evaluated by Eq. (2-45) is related to $w_{i,j,k+1/2}$ by the following equation:

$$\cos\phi_{j+1/2}\, w_{i+1/2,j+1/2,k+1/2}=\frac{1}{4}\{\cos\phi_j(w_{i,j,k+1/2}+w_{i+1,j,k+1/2})$$

$$+\cos\phi_{j+1}(w_{i,j+1,k+1/2}+w_{i+1,j+1,k+1/2})\}\,. \qquad (2\text{-}62)$$

(c) Horizontal diffusion

The finite difference analog of the horizontal diffusion term is

$$(F_T)_{i,j,k}=A_h[\{(T_{i+1,j,k}-T_{i,j,k})-(T_{i,j,k}-T_{i-1,j,k})\}/\Delta x_j^2$$
$$+\{\cos\phi_{j+1/2}(T_{i,j+1,k}-T_{i,j,k})$$
$$-\cos\phi_{j-1/2}(T_{i,j,k}-T_{i,j-1,k})\}/\cos\phi_j\,\Delta y^2]\,. \qquad (2\text{-}63)$$

(d) Vertical diffusion

The vertical diffusion term is written as

$$(G_T)_{i,j,k}=\{(DT)_{i,j,k-1/2}-(DT)_{i,j,k+1/2}\}/\Delta z_k\,, \qquad (2\text{-}64)$$

where

$$(DT)_{i,j,k+1/2}=K_h(T_{i,j,k}-T_{i,j,k+1})/\Delta z_{k+1/2}\,. \qquad (2\text{-}65)$$

T and S at time level $n+1$ are obtained from

$$\frac{T^{(n+1)}_{i,j,k}-T^{(n-1)}_{i,j,k}}{2\Delta t}=(M_T)^{(n)}_{i,j,k}+(W_T)^{(n)}_{i,j,k}+(F_T)^{(n-1)}_{i,j,k}+(G_T)^{(n-1)}_{i,j,k}+(\delta_T)^{(n+1)}_{i,j,k}\,, \qquad (2\text{-}66)$$

$$\frac{S^{(n+1)}_{i,j,k}-S^{(n-1)}_{i,j,k}}{2\Delta t}=(M_S)^{(n)}_{i,j,k}+(W_S)^{(n)}_{i,j,k}+(F_S)^{(n-1)}_{i,j,k}+(G_S)^{(n-1)}_{i,j,k}+(\delta_S)^{(n+1)}_{i,j,k}\,, \qquad (2\text{-}67)$$

where $(M_S)_{i,j,k}$, $(W_S)_{i,j,k}$, $(F_S)_{i,j,k}$ and $(G_S)_{i,j,k}$ are counterparts of $(M_T)_{i,j,k}$, $(W_T)_{i,j,k}$, $(F_T)_{i,j,k}$ and $(G_T)_{i,j,k}$, respectively. $(\delta_T)^{(n+1)}_{i,j,k}$ and $(\delta_S)^{(n+1)}_{i,j,k}$ represent time rate of change due to the convective adjustment. If the new density field, which is calculated from the temperature and salinity field without δ terms, contains a statically unstable stratification, $\rho^{(n+1)}_{i,j,k-1}>\rho^{(n+1)}_{i,j,k}$, the temperatures $T^{(n+1)}_{i,j,k-1}$ and $T^{(n+1)}_{i,j,k}$ and salinities $S^{(n+1)}_{i,j,k-1}$ and $S^{(n+1)}_{i,j,k}$ are replaced by the respective weighted mean values.

2.4.5 Boundary conditions

The lateral boundary conditions only for the western boundary ($i=2$) and southern boundary ($j=2$) are given, since the eastern boundary ($i=IM-1$) and northern boundary

$(j=JM-1)$ are treated similarly.

The condition of zero normal velocity is maintained by imposing antisymmetric conditions on the normal velocity component :

$$u^{(n)}_{3/2,j+1/2,k}=-u^{(n)}_{5/2,j+1/2,k}\,,\qquad v^{(n)}_{i+1/2,3/2,k}=-v^{(n)}_{i+1/2,5/2,k}\,. \tag{2-68}$$

For the tangential velocity component, symmetric conditions are imposed :

$$v^{(n)}_{3/2,j+1/2,k}=v^{(n)}_{5/2,j+1/2,k}\,,\qquad u^{(n)}_{i+1/2,3/2,k}=u^{(n)}_{i+1/2,5/2,k}\,. \tag{2-69}$$

In the friction terms, the no-slip condition at the western boundary is satisfied by the antisymmetric conditions :

$$u^{(n-1)}_{3/2,j+1/2,k}=-u^{(n-1)}_{5/2,j+1/2,k}\,,\qquad v^{(n-1)}_{3/2,j+1/2,k}=-v^{(n-1)}_{5/2,j+1/2,k}\,. \tag{2-70}$$

The free-slip condition at the southern boundary is satisfied by the symmetric conditions

$$u^{(n-1)}_{i+1/2,3/2,k}=u^{(n-1)}_{i+1/2,5/2,k}\,,\qquad v^{(n-1)}_{i+1/2,3/2,k}=v^{(n-1)}_{i+1/2,5/2,k}\,. \tag{2-71}$$

On the temperature and salinity, the following symmetric conditions are imposed both in the advection and diffusion terms:

$$T^{(n)}_{1,j,k}=T^{(n)}_{3,j,k}\,,\quad T^{(n-1)}_{1,j,k}=T^{(n-1)}_{3,j,k}\,,\quad T^{(n)}_{i,1,k}=T^{(n)}_{i,3,k}\,,\quad T^{(n-1)}_{i,1,k}=T^{(n-1)}_{i,3,k}\,,$$
$$S^{(n)}_{1,j,k}=S^{(n)}_{3,j,k}\,,\quad S^{(n-1)}_{1,j,k}=S^{(n-1)}_{3,j,k}\,,\quad S^{(n)}_{i,1,k}=S^{(n)}_{i,3,k}\,,\quad S^{(n-1)}_{i,1,k}=S^{(n-1)}_{i,3,k}\,. \tag{2-72}$$

The conditions at the ocean bottom are, referring to Eqs. (2-51) and (2-65), satisfied by setting as follows:

$$w^{(n)}_{i,j,KM+1/2}=w^{(n)}_{i+1/2,j+1/2,KM+1/2}=0\,,$$
$$(Du)^{(n-1)}_{i+1/2,j+1/2,KM+1/2}=(\tau^{\lambda}_{B})^{(n-1)}_{i+1/2,j+1/2}/\rho_0\,,$$
$$(Dv)^{(n-1)}_{i+1/2,j+1/2,KM+1/2}=(\tau^{\phi}_{B})^{(n-1)}_{i+1/2,j+1/2}/\rho_0\,,$$
$$(DT)^{(n-1)}_{i,j,KM+1/2}=(DS)^{(n-1)}_{i,j,KM+1/2}=0\,. \tag{2-73}$$

At the ocean surface, the following conditions are imposed :

$$w^{(n)}_{i,j,1/2}=w^{(n)}_{i+1/2,j+1/2,1/2}=0\,,$$
$$(Du)^{(n-1)}_{i+1/2,j+1/2,1/2}=(\tau^{\lambda})^{(n-1)}_{i+1/2,j+1/2}/\rho_0\,,$$
$$(Dv)^{(n-1)}_{i+1/2,j+1/2,1/2}=(\tau^{\phi})^{(n-1)}_{i+1/2,j+1/2}/\rho_0\,, \qquad (2\text{-}74)$$
$$(DT)^{(n-1)}_{i,j,1/2}=(Q_T)^{(n-1)}_{i,j}/\rho_0 c\,,$$
$$(DS)^{(n-1)}_{i,j,1/2}=(Q_S)^{(n-1)}_{i,j}/\rho_0\,.$$

2.4.6 Vorticity equation

The finite difference analog of Eq. (2-28) for the stream function is

$$\begin{aligned}L(q_{i,j})&=ZTD_{i,j}\\&=\frac{1}{2\Delta x_j}(VM_{i+1/2,j+1/2}+VM_{i+1/2,j-1/2}-VM_{i-1/2,j+1/2}-VM_{i-1/2,j-1/2})\\&\quad-\frac{1}{2\cos\phi_j\Delta y}\{\cos\phi_{j+1/2}(UM_{i-1/2,j+1/2}+UM_{i+1/2,j+1/2})\\&\qquad-\cos\phi_{j-1/2}(UM_{i-1/2,j-1/2}+UM_{i+1/2,j-1/2})\}\,,\end{aligned} \qquad (2\text{-}75)$$

where L denotes the finite difference analog of the Laplacian operator ∇^2,

$$\begin{aligned}L(q_{i,j})&=\frac{1}{\Delta x_j^2}(q_{i+1,j}+q_{i-1,j}-2q_{i,j})\\&\quad+\frac{1}{\cos\phi_j\Delta y^2}\{\cos\phi_{j+1/2}(q_{i,j+1}-q_{i,j})-\cos\phi_{j-1/2}(q_{i,j}-q_{i,j-1})\}\,,\end{aligned} \qquad (2\text{-}76)$$

and

$$q_{i,j}=\frac{\Psi^{(n+1)}_{i,j}-\Psi^{(n-1)}_{i,j}}{2\Delta t H}\,,$$

$$UM_{i+1/2,j+1/2}=\frac{1}{H}\sum_{k=1}^{KM}(U^{(n,n-1)}_{i+1/2,j+1/2,k}+2\Omega\sin\phi_{j+1/2}\cdot v^{(n)}_{i+1/2,j+1/2,k})\Delta z_k\,, \qquad (2\text{-}77)$$

$$VM_{i+1/2,j+1/2}=\frac{1}{H}\sum_{k=1}^{KM}(V^{(n,n-1)}_{i+1/2,j+1/2,k}-2\Omega\sin\phi_{j+1/2}\cdot u^{(n)}_{i+1/2,j+1/2,k})\Delta z_k\,.$$

The difference equation (2-76) is solved together with the boundary conditions, $q_{i,j}=0$ along the lateral boundaries.

In the present model, the following Fourier direct method (Williams, 1969) is used to solve the Poisson equation. Let

$$q_{i,j}=\sum_{l=3}^{IM-2}Q_{l,j}\sin\left\{\frac{(l-2)(i-2)}{IM-3}\pi\right\}, \qquad (2\text{-}78)$$

then Eq. (2-75) may be written after a little manipulation as

$$\frac{\cos\phi_{j-1/2}}{R_j\cos\phi_j\Delta y^2}Q_{l,j-1}+\left[\frac{2}{R_j\Delta x_j^2}\cos\left\{\frac{(l-2)}{IM-3}\pi\right\}-1\right]Q_{l,j}+\frac{\cos\phi_{j+1/2}}{R_j\cos\phi_j\Delta y^2}Q_{l,j+1}$$

$$=\frac{2}{IM-3}\sum_{i=3}^{IM-2}\sin\left\{\frac{(l-2)(i-2)}{IM-3}\pi\right\}\frac{ZTD_{i,j}}{R_j}, \tag{2-79}$$

where

$$R_j=\frac{2}{\Delta x_j^2}+\frac{\cos\phi_{j-1/2}+\cos\phi_{j+1/2}}{\cos\phi_j\Delta y^2}. \tag{2-80}$$

The equation (2-79) is solved for the Fourier coefficient $Q_{i,j}$. Then the stream function tendency $q_{i,j}$ can be obtained from $Q_{i,j}$ through Eq. (2-78).

2.4.7 Programming

The computer program for the model is coded according to the program flow of Semtner (1974), and changed to take advantage of the array processor.

To check the coding of the program, volume integrals over the entire basin of the terms in the temperature and salinity equations (2-66) and (2-67) are taken on a certain time step. The integrals of the advective terms and of the horizontal diffusion terms should be essentially zero. The integrals of the vertical diffusion terms should also be zero except the surface flux terms. Thus the time changes of the volume integrals of temperature and salinity must be equal to the surface fluxes to within truncation error.

A further check is made as to the energy balance. The time change of the volume averaged kinetic energy of the vertical shear current is derived from Eqs. (2-29) and (2-30) as

$$\frac{1}{V}\int_V\frac{\partial}{\partial t}\left(\frac{u'^2+v'^2}{2}\right)dm=\frac{1}{V}\int_V(u'U+v'V)dm=\sum_{i=1}^{7}E_i', \tag{2-81}$$

where V denotes the total volume of the model ocean. For the time change of the kinetic energy of the vertical mean current, the following equation is derived from Eq. (2-28):

$$\frac{1}{S}\int_S\frac{\partial}{\partial t}\left(\frac{\bar{u}^2+\bar{v}^2}{2}\right)ds=-\frac{1}{V}\int_S\Psi ZTDds$$

$$=-\frac{1}{V}\int_S\Psi\left\{\frac{1}{a\cos\phi}\frac{\partial\bar{V}}{\partial\lambda}-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\cdot(\cos\phi\,\bar{U})\right\}ds$$

$$=\sum_{i=1}^{7}\bar{E}_i, \tag{2-82}$$

where S denotes the area of the ocean and ZTD the righthand side of Eq. (2-28). E'_i and $\bar{E}_i$ represent the contributions associated with the horizontal pressure gradient E'_1 and $\bar{E}_1$, horizontal advection E'_2 and $\bar{E}_2$, vertical advection E'_3 and $\bar{E}_3$, horizontal diffusion E'_4 and $\bar{E}_4$, vertical diffusion E'_5 and $\bar{E}_5$, wind stress E'_6 and $\bar{E}_6$, and bottom friction E'_7 and $\bar{E}_7$, respectively. The contribution associated with the metric terms is included in E'_2 and $\bar{E}_2$. $\bar{E}_1$ and $\bar{E}_5$ should be zero. The equality (2-81) must hold except for a small residual, which comes from the semi-implicit treatment of the Coriolis terms. Furthermore, the following relations which represent the transformation of kinetic energy from the vertical shear component to the vertically uniform component and the transformation of potential energy to shear kinetic energy must hold:

$$E'_2+E'_3=\frac{1}{V}\int_V\Big\{u'\Big(-\frac{u}{a\cos\phi}\frac{\partial u}{\partial\lambda}-\frac{v}{a}\frac{\partial u}{\partial\phi}-w\frac{\partial u}{\partial z}+\frac{uv}{a}\tan\phi\Big)$$

$$+v'\Big(-\frac{u}{a\cos\phi}\frac{\partial v}{\partial\lambda}-\frac{v}{a}\frac{\partial v}{\partial\phi}-w\frac{\partial v}{\partial z}-\frac{u^2\tan\phi}{a}\Big)\Big\}dm$$

$$=-\frac{1}{V}\int_V\Big\{\bar{u}\Big(-\frac{u}{a\cos\phi}\frac{\partial u}{\partial\lambda}-\frac{v}{a}\frac{\partial u}{\partial\phi}-w\frac{\partial u}{\partial z}+\frac{uv}{a}\tan\phi\Big)$$

$$+\bar{v}\Big(-\frac{u}{a\cos\phi}\frac{\partial v}{\partial\lambda}-\frac{v}{a}\frac{\partial v}{\partial\phi}-w\frac{\partial v}{\partial z}-\frac{u^2\tan\phi}{a}\Big)\Big\}dm$$

$$=-(\bar{E}_2+\bar{E}_3)\,, \qquad (2\text{-}83)$$

$$E'_1=\frac{1}{V}\int_V\Big\{u'\Big(-\frac{1}{\rho_0 a\cos\phi}\frac{\partial}{\partial\lambda}\int_z^0\rho g dz\Big)+v'\Big(-\frac{1}{\rho_0 a}\frac{\partial}{\partial\phi}\int_z^0\rho g dz\Big)\Big\}dm$$

$$=\frac{1}{V}\int_V-\frac{\rho g w}{\rho_0}dm\,. \qquad (2\text{-}84)$$

2.4.8 Notes

(1) To maintain the no-slip boundary condition at the western and eastern walls, the antisymmetric conditions are imposed on the horizontal velocity in the friction terms. Nevertheless, the symmetric conditions are imposed on the tangential velocity component in the advection terms. Otherwise the mass may not be conserved. This is because the boundary condition on the vertically integrated current (i. e., Ψ =constant along the lateral walls) guarantees no normal flow, but does not mean no tangential flow.

(2) The boundary conditions on vertical velocity — $w=0$ at the ocean surface and bottom

— can not be thoroughly satisfied at the same time. Let $w=0$ at the ocean surface, then w at the bottom is calculated from Eq. (2-45) or Eq. (2-59). In the present calculations, vertical velocities at the bottom are less than 10^{-4} of those in the interior. Therefore, the condition at the bottom is satisfied to within truncation error.

2.5 External forcing

The model ocean is driven by wind stress and heat and salinity fluxes through the sea surface. The forcing functions used to obtain a steady state are steady in time and constant in longitude.

The wind stress has no meridional component, $\tau^{\phi}=0$. The zonal component τ^{λ} is taken from the annual mean zonal wind stress for the Pacific Ocean given in Wyrtki and Meyers (1976) for 30°S−30°N and Kutsuwada and Sakurai (1982) for 30°N−54°N (Fig. 2-5 (a)). The tropical westward stress is minimum at 1°N.

The thermal forcing is given by the approximate formula proposed by Haney (1971). The heat flux through the surface is calculated from

$$Q_T(\lambda,\ \phi,\ t)=Q\{T_a{}^*(\phi)-T_1(\lambda,\ \phi,\ t)\}, \tag{2-85}$$

where $T_a{}^*$ is apparent atmospheric equilibrium temperature,

$$T_a{}^*(\phi)=13.0+17.0\cos(\frac{\phi}{40}\frac{\pi}{2}), \tag{2-86}$$

(Fig. 2-5 (b)), T_1 the calculated temperature of the top layer of the model ocean, and Q a coupling coefficient. In this study, Q is taken to be a constant of 50 cal/cm²•day•K. This value indicates that the temperature of the uppermost layer (60 m) is adjusted to $T_a{}^*$ on a time scale of about 120 days.

The salinity flux through the surface is calculated from

$$Q_S(\lambda,\phi,t)=S_1(\lambda,\phi,t)\{E(\phi)-P(\phi)\}, \tag{2-87}$$

where S_1 is the calculated salinity of the top layer, E the evaporation, and P the precipitation. $E(\phi)$ is taken from the zonal mean annual evaporation for the Pacific Ocean estimated by Wyrtki (1965), Weare et al. (1981), and Saiki (private communication, 1981). $P(\phi)$ is taken from the zonal mean annual precipitation for the Pacific Ocean estimated by Dorman and Bourke (1979). The meridional profile of $(P-E)(\phi)$, with addition of some constant value for zero total $(P-E)$, is shown in Fig. 2-5 (c).

2.6 Vertical resolution of the model ocean and initial conditions

The model ocean is divided into eight layers in the vertical, with boundaries at 0, 60, 190, 380, 590, 850, 1450, 2700, and 5000 m. The depth of the levels z_k is given in Table 2-1.

The initial state of the model is a horizontally uniform stratification with no motion. The vertical distributions of temperature and salinity are given in Table 2-1. Except for the upper two levels, they are taken from hydrographic data in the western tropical North

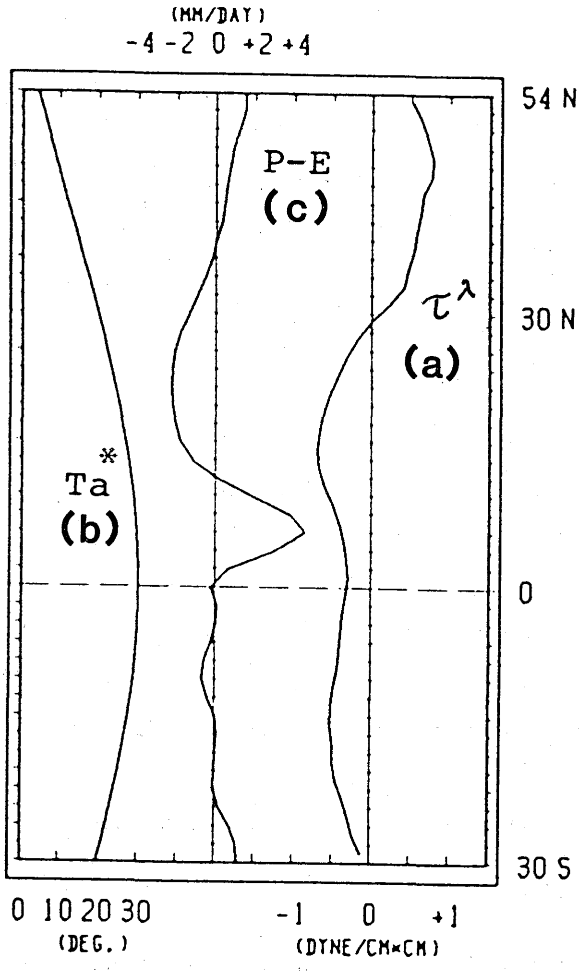

Fig. 2-5 The prescribed external forcing function: (a) the zonal wind stress τ^{λ}, (b) the apparent atmospheric equilibrium temperature Ta^{*}, and (c) the precipitation minus evaporation ($P-E$).

Pacific. Fig. 2-6 shows the corresponding density profile.

2.7 Computation and parameters

The convergence of the solution to an equilibrium is accelerated by two ways, in addition to the numerical and programming techniques mentioned in the previous sections. One way

Table 2-1 Depth of levels z_k and initial values of temperature and salinity.

No.	Depth (m)	T (°C)	S (‰)
1	20	9.2	34.515
2	100	9.1	34.52
3	280	9.0	34.525
4	480	7.2	34.54
5	700	5.9	34.52
6	1000	4.55	34.545
7	1900	2.35	34.62
8	3500	1.55	34.68

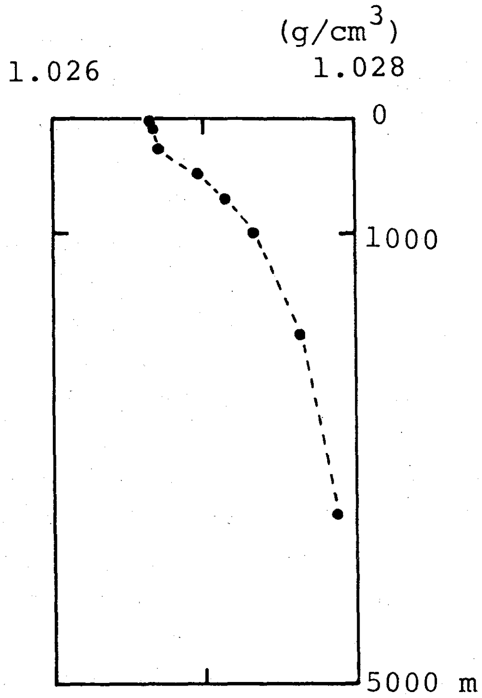

Fig. 2-6 Initial density stratification.

is to separate the integration into two stages, stage I and stage II, where the zonal grid spacing in stage I is twice as large as that in stage II. The other way is to use $\Delta t/\alpha$ ($\alpha>1$) as the time step for integration of the barotropic vorticity equation. The latter treatment is equivalent to taking a shorter time step for the rapidly adjusting barotropic field than for the baroclinic field. This is justified when all local time derivatives vanish.

The values of all parameters used in the model are given in Table 2-2. The magnitude of A_m is determined so that the frictional width of the western boundary current is marginally resolved by the zonal grid spacing $\Delta\lambda$ (Takano, 1974). For A_h, a much smaller value is chosen than that required for A_m, because the horizontal diffusion and surface flux will nearly balance if the same value is chosen, especially in stage I. But a further decrease of A_h excites a computational mode.

As noted above, external gravity waves are filtered out by employing the rigid-lid assumption, and inertial oscillations are handled by treating the Coriolis terms implicitly. Hence, the maximum time step which can be used in this system is determined approximately

Table 2-2 Values of parameters used in the model.

parameters	stage I	stage II	
a	6375 km		radius of the earth
Ω	7.292×10^{-5} sec^{-1}		rotation rate of the earth
g	980 cm/sec^2		acceleration of gravity
ρ_0	1.025 g/cm^3		reference density of sea water
c_p	($\rho_0 c_p=1.0$ cal/cm$^3\cdot$K)		specific heat of sea water
KM	8		number of levels in vertical
H	5000 m		total ocean depth
JM	45		number of T, S points in latitude
$\Delta\phi$	2.0°		meridional grid separation
IM	23	43	number of T, S points in longitude
$\Delta\lambda$	5.0°	2.5°	zonal grid separation
K_m	1.0 cm^2/sec		vertical eddy viscosity
K_h	1.0 cm^2/sec		vertical eddy conductivity
A_m	2.0×10^9, 3.0×10^8 cm^2/sec		horizontal eddy viscosity
A_h	2.0×10^7 cm^2/sec		horizontal eddy conductivity
Δt	4.8	4.0 hr	time step
α	10		($\Delta t/\alpha$: time step for the vorticity equation)
Q	50 cal/cm$^2\cdot$day$\cdot$K		coupling coefficient

by the phase speed of internal gravity waves. We set $\alpha=10$. If $\alpha=1$, that is, if the same time step is taken for the vertical mean current as for the vertical shear current, Δt must be reduced to about one hour in order to suppress computational instability.