9. Convective adjustment and condensation*

Latent heat is released within the model atmosphere in three different ways. They are (1) precipitation due to large-scale condensation, (2) cumulus precipitation and (3) middle level convective precipitation. Description about the second process is given in Chapter 7. The remaining two are described in this chapter.

In the model, adjustment processes are taken into consideration in the following order. First, dry convective adjustment; second, moist convective adjustment (except for penetrative cumulus convection); third, large-scale condensation; and finally, penetrative cumulus convection. The flow diagram is shown in Fig. 9.1. In their definitions, "penetrative cumulus convection" means a convection which has its origin in the planetary boundary layer (PBL); "middle level convection", a convection which has its origin in the free atmosphere. "Large-scale condensation" means the condensation associated with a supersaturation at the grid point. "Dry convective adjustment" occurs when the air becomes unstable dry statically. This

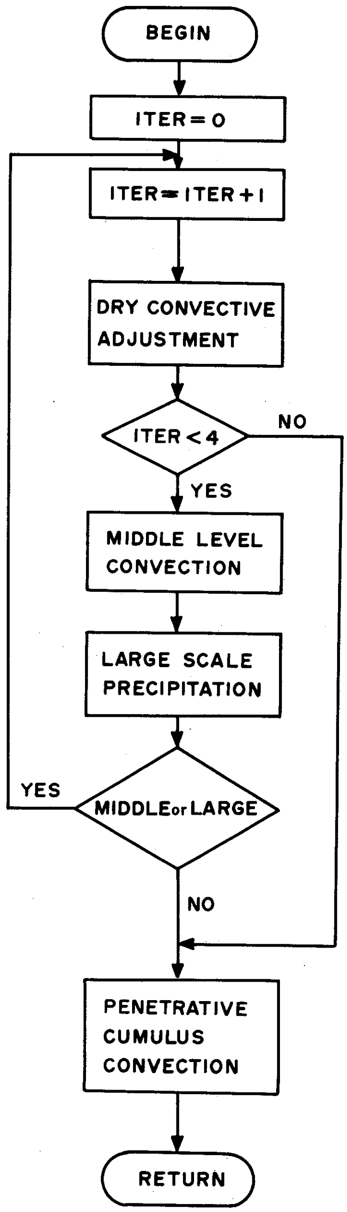

Fig. 9.1 Flow diagram of convective adjustment.

* This chapter is prepared by A. Kitoh: Forecast Research Division

process does not accompany any condensation.

9.1 Some definitions of variables

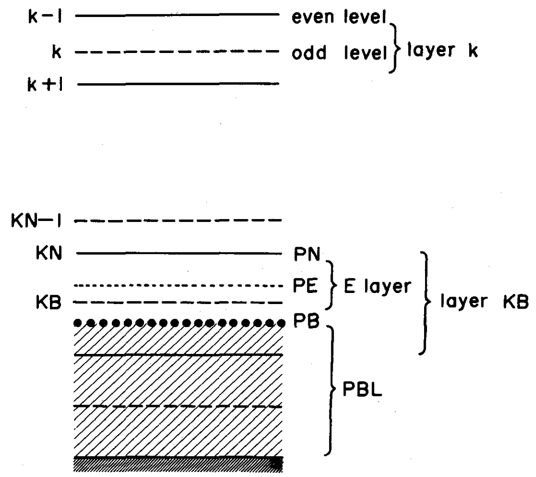

Fig. 9.2 Vertical indices.

The definition of vertical layers in this chapter is found in Fig. 9.2. T, q and z are temperature, the water vapor mixing ratio, and the geopotential height. We define the following environmental variables at level k:

Dry static energy

$$s_k = c_p T_k + g z_k; \tag{9.1}$$

Moist static energy

$$h_k = s_k + L q_k; \tag{9.2}$$

Saturation moist static energy

$$h_k^* = s_k + L q_k^*; \tag{9.3}$$

Cloud dry static energy at the vanishing buoyancy level

$$\hat{s}_k = s_k - \frac{L \varepsilon_k \delta}{1 + \gamma_k \varepsilon_k \delta}(q_k^* - q_k); \tag{9.4}$$

Cloud moist static energy at the vanishing buoyancy level

$$\hat{h}_k = h_k - \frac{(1 + \gamma_k) L \varepsilon_k \delta}{1 + \gamma_k \varepsilon_k \delta}(q_k^* - q_k), \tag{9.5}$$

c_p is the specific heat at constant pressure, g is the accerelation of gravity, L is the latent heat of condensation, * is the saturated value, ^ is the even level value, $\gamma = \frac{L}{c_p}\left(\frac{\partial q^*}{\partial T}\right)$, $\varepsilon = \frac{c_p T}{L}$ and $\delta = 0.609$.

The saturation mixing ratio is given by

$$q^*(T) = \frac{M_w}{M_d} \cdot \frac{e_s(T)}{p - e_s(T)}, \tag{9.6}$$

where M_w and M_d are the mean molecular weight of water vapor and dry air, respectively ($M^w/M_d = 0.622$), and p is the pressure. The saturation vapor pressure e_s is given by Teten's equation

$$e_s(T) = e_0 10^{\frac{at}{b+t}}, \tag{9.7}$$

where $t=T-273.1$ is the temperature in Celsius, $a=7.5$, $b=237.3$, and e_0 ($=6.11$mb) is the vapor pressure at $T=273.1$K. The saturation mixing ratio gradient is given by

$$\left(\frac{\partial q^*}{\partial T}\right)_{T=T_\nu}=B_e\frac{q^*(T_\nu)}{T_\nu^2}, \tag{9.8}$$

using the another approximate relation

$$e_s(T)=e_1\exp(A_e-B_e/T), \tag{9.9}$$

where $e_1=1$ mb, $A_e=21.656$, and $B_e=5417.98$K.

We define E-layer as the layer located between the top of the PBL and the upper boundary of the layer KB. We define s and h in the E-layer as s_E and h_E in the same manner.

9.2 Dry convective adjustment

If the model atmosphere is found to be dry-adiabatically unstable after the advective process, *i.e.*,

$$\theta_k<\theta_{k+2} \quad \text{for any odd k,} \tag{9.10}$$

subgrid scale dry convection is assumed to occur, bringing the layer back to a dry adiabatic lapse rate. This adjustment is done between adjacent two layers in the order of increasing k. If the condition (9.10) is satisfied in more than two layers in contact, we adjust those layers collectively. See Fig. 9.3.

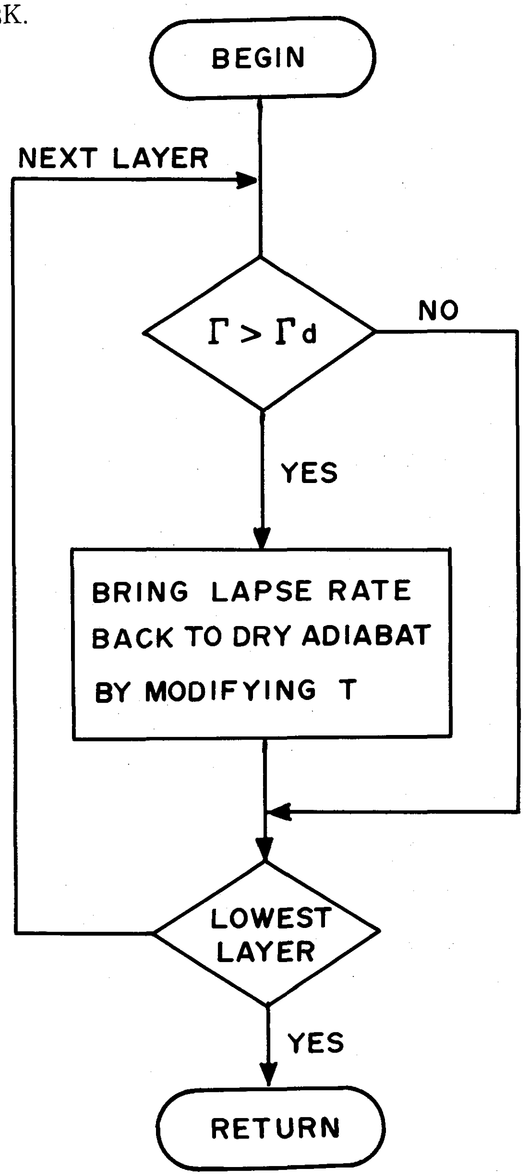

Fig. 9.3 Flow diagram of dry convective adjustment.

In adjusting layers from k=KBEGIN to k=KEND, the new temperatures T'_k are determined in the following way:

$$\sum_{k=KBEGIN}^{KEND}{}' T'_K\Pi_K\Delta\sigma_k=\sum_{k=KBEGIN}^{KEND}{}' T_K\Pi_K\Delta\sigma_k, \tag{9.11}$$

$$\theta'_{\mathrm{KBEGIN}} = \theta'_{\mathrm{KBEGIN}+2} = \cdots = \theta'_{\mathrm{KEND}}, \tag{9.12}$$

where Σ' is the summation over odd k's only. Primed variables indicate those after adjustment. Using the definition of θ_k,

$$\theta_k = T_k (p_k)^{-\kappa} \tag{9.13}$$

, (9.11) and (9.12), adjusted potential temperature θ_{new} is given by

$$\theta_{\mathrm{new}} = \frac{\sum'^{\mathrm{KEND}}_{k=\mathrm{KBEGIN}} T_K \Pi_K \Delta\sigma_K}{\sum'^{\mathrm{KEND}}_{k=\mathrm{KBEGIN}} (p_k)^{\kappa} \Pi_K \Delta\sigma_K} \tag{9.14}$$

Then,

$$T'_k = \theta_{\mathrm{new}} (p_k)^{\kappa} \text{ for KBEGIN} \leq k \leq \text{KEND}. \tag{9.15}$$

9.3 Middle level convection

In this section moist convective adjustment which has a root in the free atmosphere is treated. The flow chart is shown in Fig. 9.4. The moist convective adjustment occurs whenever either

$$h_{k+2} > \hat{h}^*_k, \tag{9.16}$$

for any odd k ($\leq K-2$) or

$$h_E > \hat{h}^*_{KN-1}. \tag{9.17}$$

The moist static energy of a saturated non-buoyant air parcel is given by

$$\hat{h}^*_k = h^*_k - \frac{(1+\gamma_k) L \varepsilon_k \delta}{1+\gamma_k \varepsilon_k \delta} (q^*_k - q_k). \tag{9.18}$$

If $h_{k+2} > \hat{h}^*_k$, an air parcel rising from the lower level will have positive buoyancy at the upper level k. At present we

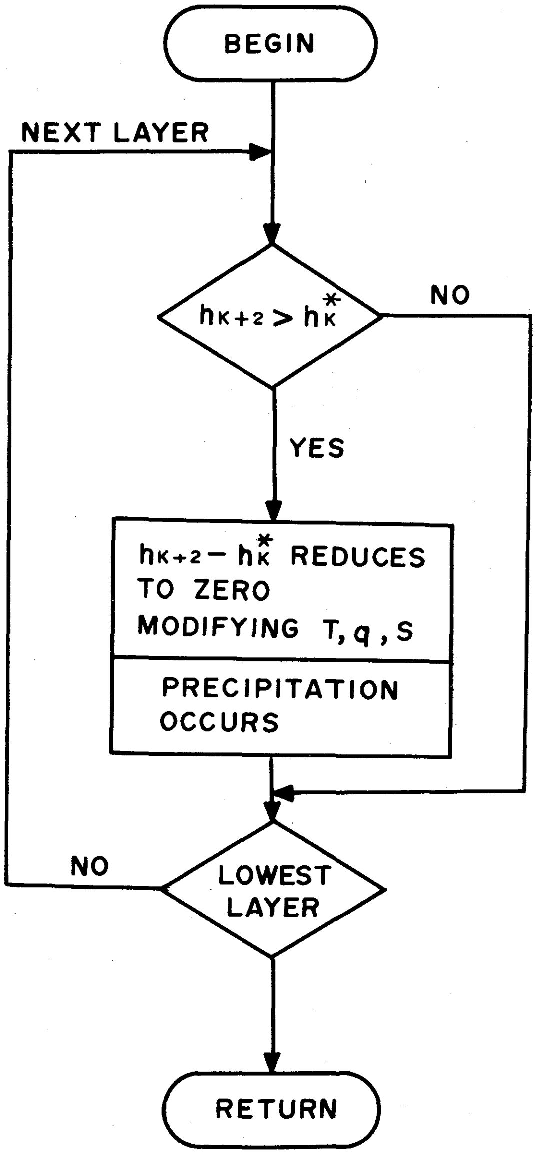

Fig. 9.4 Flow diagram of middle level convection.

approximate $\hat{h}_k^*$ by h_k^*, so when $h_k^* < h_{k+2}$ for any odd k ($\leq K-2$), we assume that moist convection occurs in such a way as to reduce $h_{k+2} - h_k^*$ to zero.

The mixing ratio of cloud air which detrains at the upper level is given by

$$(q_c)_k = q_k^* + \frac{\gamma_k}{1+\gamma_k} \cdot \frac{1}{L}(h_{k+2} - h_k^*) \tag{9.19}$$

If $(q_c)_k > q_{k+2}$, we adjust $(q_c)_k$ to q_{k+2}.

The moisture budget for the two layers are

$$\frac{\Delta p_k}{g}\frac{\partial q_k}{\partial t} = \eta[(q_c)_k - q_{k+1}], \tag{9.20}$$

$$\frac{\Delta p_{k+2}}{g}\frac{\partial q_{k+2}}{\partial t} = \eta[q_{k+1} - q_{k+2}], \tag{9.21}$$

where η is the mass flux at level $k+1$.

The dry static energy budget for the two layers are

$$\frac{\Delta p_k}{g}\frac{\partial s_k}{\partial t} = \eta\left[\frac{1}{1+\gamma_k}(h_{k+2} - h_k^*) + s_k - s_{k+1}\right], \tag{9.22}$$

$$\frac{\Delta p_{k+2}}{g}\frac{\partial s_{k+2}}{\partial t} = \eta[s_{k+1} - s_{k+2}]. \tag{9.23}$$

The first term on the right hand side of eq. (9.22) represents the detrainment of cloud air into the layer; the second term represents adiabatic warming due to the subsidence in the environment. At present we assume that the temperature change does not exceed 1.5 K per step.

From (9.21) and (9.23), we obtain

$$\frac{\Delta p_{k+2}}{g}\frac{\partial h_{k+2}}{\partial t} = \eta(h_{k+1} - h_{k+2}), \tag{9.24}$$

and from (9.22)

$$\frac{\Delta p_k}{g}\frac{\partial h_k^*}{\partial t} = \eta[h_{k+2} - h_k^* + (1+\gamma_k)(s_k - s_{k+1})], \tag{9.25}$$

with the aid of

$$\frac{\partial h_k^*}{\partial t} = (1+\gamma_k)\frac{\partial s_k}{\partial t}. \tag{9.26}$$

From (9.24) and (9.25),

$$\frac{\partial}{\partial t}(h_{k+2} - h_k^*) = \eta g\left[\frac{1}{\Delta p_{k+2}}(h_{k+1} - h_{k+2}) - \frac{1}{\Delta p_k}\{(h_{k+2} - h_k^*) + (1+\gamma_k)(s_k - s_{k+1})\}\right].$$

(9.27)

If we assume an adjustment time τ, mass flux becomes

$$\eta=\frac{1}{\tau g}\frac{h_{k+2}-h_k^*}{\frac{1}{\Delta p_{k+2}}(h_{k+1}-h_{k+2})+\frac{1}{\Delta p_k}\{(h_{k+2}-h_k^*)+(1+\gamma_k)(s_k-s_{k+1})\}}. \tag{9.28}$$

Currently we use $\tau=\Delta t_d=60$ minutes. Finally from (9.20) and (9.21), precipitation due to middle level convection is given by

$$P_{ML}=\eta[q_{k+2}-(q_c)_k]\Delta t_d. \tag{9.29}$$

9.4 Large-scale condensation

Large-scale condensation occurs if the grid cell is supersaturated, *i.e.*, q_k is greater than q_k^* where q_k is the water vapor mixing ratio and q_k^* is the saturation mixing ratio at the temperature T_k and the pressure p_k (Fig. 9.5).

This condensation removes moisture from the atmosphere and warms the atmosphere by releasing latent heat, with the warming in turn modifying the saturation mixing ratio. The condensation proceeds until $q_k=q_k^*(T_k)$ at the new temperature. When condensation occurs in a layer which is not the lowermost layer of the model, the condensed water is brought into the next lower layer, and is forced to evaporate. This process is repeated until the lowest layer is reached. When the lowest layer is saturated, the condensed water precipitates onto the ground either as rain or snow according to the surface air temperature.

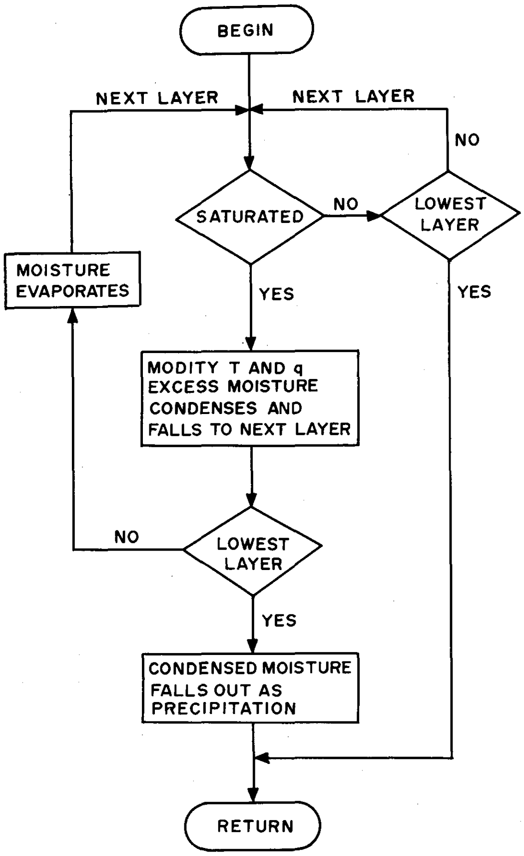

Fig. 9.5 Flow diagram of large-scale condensation.

Let $C\Delta t$ denote the amount of condensation at level k per unit mass of dry air, when $q_k > q_k^*$. Then

$$q_k' = q_k - C\Delta t \tag{9.30}$$

$$T_k' = T_k + \frac{L}{c_p} C\Delta t, \tag{9.31}$$

$$q_k' = q^* \ (T = T_k',\ p = p_k). \tag{9.32}$$

where the primes denote the modified values due to condensation. (9.32) describes the saturation condition for the modified moisture and temperature. From these three equations, we may obtain an equation for the modified temperatures;

$$q_k - \frac{c_p}{L}(T_k' - T_k) = q^* \ (T = T_k',\ p = p_k). \tag{9.33}$$

With q_k, T_k, p_k and the functional form of $q^*(T,p)$ given, the transcendental equation (9.33) can be solved iteratively for T_k' by Newton's method. After T_k' is obtained, we can calculate $C\Delta t$ and q_k' from (9.30) and (9.31).

Fig. 9.6 schematically shows that the saturation mixing ratio q^* as a function of temperature denoted by

$$q = q^*(T) \tag{9.34}$$

and also a segment of the line,

$$q = q_0 - \frac{c_p}{L}(T - T_0) \tag{9.35}$$

which passes the point $A_0(T_0, q_0)$ in the (T, p) plane. The intersection of the saturation curve, $q = q^*(T)$, and the line given by (9.35) gives the solution of (9.33). This point is denoted by $A(T_k', q_k')$ in the figure.

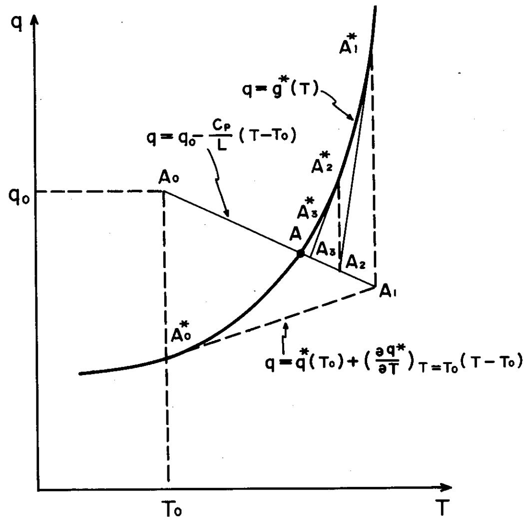

Fig. 9.6 Schematical explanation of adjustment in large-scale condensation.

We may obtain the intersection point approximately by the iterative method. Let $A_1, A_2, \cdots$ be a sequence of points whose coordinates in the (T,q) plane are, respectively, (T_1, q_1), $(T_2, q_2), \cdots$. Such a sequence can be generated by the use of tangential lines to the saturation curve at T_ν, for $\nu = 0, 1, 2, \cdots$,

$$q = q^*(T_\nu) + \left(\frac{\partial q^*}{\partial T}\right)_{T=T_\nu}(T - T_\nu). \tag{9.36}$$

Choosing

$$T_0 = T_k \text{ and } q_0 = q_k, (\nu = 0), \tag{9.37}$$

we determine $(T_{\nu+1}, q_{\nu+1})$ recursively by

$$q_{\nu+1} = q^*(T_\nu) + \left(\frac{\partial q^*}{\partial T}\right)_{T=T_\nu} (T_{\nu+1} - T_\nu), \tag{9.38}$$

$$q_{\nu+1} = q_\nu - \frac{c_p}{L}(T_{\nu+1} - T_\nu), \tag{9.39}$$

for $\nu \geq 0$. From the slope of the saturation curve, we see that the point A_ν uniformly approaches A as ν increases, and that for ever $\nu \geq 1$,

$$q_\nu < q^*(T_\nu). \tag{9.40}$$

Thus, even the first iteration leads to an unsaturated condition and higher accuracy is obtained with increasing ν. At present, we take

$\nu_{max} = 1$ for the upper layers,

$\nu_{max} = 3$ for the lowest layer and for the E- and the P- layer

where ν_{max} is the maximum number of iteration in the layer.

From (9.38) and (9.39),

$$C_{\nu+1}\Delta t = \frac{c_p}{L}(T_{\nu+1} - T_\nu) = \frac{q_\nu - q^*(T_\nu)}{1 + \frac{L}{c_p}\left(\frac{\partial q^*}{\partial T}\right)_{T=T_\nu}}, \tag{9.41}$$

$$T_{\nu+1} = T_\nu + \frac{L}{c_p} C_{\nu+1}\Delta t, \tag{9.42}$$

$$q_{\nu+1} = q_\nu - C_{\nu+1}\Delta t. \tag{9.43}$$

In summary,

$$T'_k = T_k + \frac{L}{c_p}\sum_{\nu=1}^{\nu max} C_\nu \Delta t, \tag{9.44}$$

$$q'_k = q_k - \sum_{\nu=1}^{\nu max} C_\nu \Delta t, \tag{9.45}$$

$$T'_{k+2} = T_{k+2} - \frac{L}{c_p}\sum_{\nu=1}^{\nu max} C_\nu \Delta t\ \Pi_k \Delta\sigma_k / (\Pi_{k+2}\ \Delta\sigma_{k+2}), \tag{9.46}$$

$$q'_{k+2} = q_{k+2} + \sum_{\nu=1}^{\nu max} C_\nu\ \Delta t\ \Pi_k \Delta\sigma_k / (\Pi_{k+2}\ \Delta\sigma_{k+2}), \tag{9.47}$$

where C_ν is determined by (9.41), (9.42), (9.43). At the lowest layer the condensation reaches the earth's surface and the large-scale precipitation per unit area becomes

$$P_{LS}=\frac{\Pi}{g}\sum_{\nu=1}^{\nu max} C_\nu \Delta t \cdot \Delta\delta_K \tag{9.48}$$

where $\Delta\delta_K$ is the depth of the lowest layer in δ-coordinate, $\Pi=p_S-p_I$, p_S is the pressure at the earth's surface and $p_I=100$mb.