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An Estimation of the Heat Flux and the Water Vapour Flux by the Bulk Aerodynamic Method

by

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Abstract

Introducing a modified integral diffusivity whose integral range does not contain the surface values of temperature and water vapour, the bulk aerodynamic method is surveyed by the use of the data obtained on the joint exploration, which was carried out at the Chiba seaport in 1966 by the Meteorological Research Institute, the observational section, JMA and the Meteorological College.

Comparisons of the estimation of H and E due to the profile method with that due to the bulk method are made and the dependence of the modified integral diffusivity on the stability ratio is also sought. Finally the empirical bulk equations for the estimation of H and E without using the surface values are presented.

1. Introduction

From the recent developments in numerical forecasting, it has become necessary to account for effects of heat coming from the surface of either ground or ocean, but it seems that there is a gap between the classical Jacob's equation and the established turbulent heat transfer equation near the ground. To fill in this gap, attention has been concentrated on cumulus convection, but there is also the bulk aerodynamic method in a modified version using the data obtained from the meteorological network.

For this purpose, we first reconsider the bulk aerodynamic method with the modified integral diffusivity to estimate the eddy heat flux H and the water vapor flux E only from the ground surface.

Many reports and a good summary on the bulk aerodynamic method have already been issued (for example, DEACON and WEBB, 1962; BUDYKO, 1956; BUDOGOVSKY, 1964; ROBINSON, 1966), but it is still necessary to know the accurate values of temperature and water vapour on the surface of land or sea, because these values are hardly estimated by the data in the meteorological synoptic network. Therefore, in this paper an attempt is made, after introducing the modified integral diffusivety, to establish a bulk method in which observational values of temperature and water vapour only at two levels of height except at the surface and values of wind speed at one level of height, are required to estimate H and E. Our data are obtained in the joint observation at the flat embedded field in the China seaport inside Tokyo Bay, in October and November, 1966. This joint observation was carried out in cooperation with members of the observational section, the Research Institute and the Meteorological College, JMA.

2. Bulk aerodynamic method

The vertical flux F_S of a physical quaantity S may be expressed as follows with K_S, the eddy diffusivity of S and K_M, the eddy diffusivity of momentum:

$$F_s = -\frac{K_s}{K_m}\frac{\partial S/\partial z}{\partial U/\partial z} \tag{1}$$

by dividing the vertical flux equation of S by the corresponding vertical flux equation for momentum. Furthermore, assuming similarity between the profiles of S and of wind speed, the general expression for F_s may be written as

$$F_s = f\left(\frac{K_s}{K_M}, \frac{z}{L}\right)\rho(s_0 - s_z)U_z \tag{2}$$

where L is Monin-Obukhov's length (stability length), U is the mean wind speed at a height of z and ρ is air density, and S_0, S_z are values at the ground surface and a height of z, respectively. The general expression can be written, respectively, for momentum,

$$\tau = C_d \rho U^2{}_z \tag{3}$$

for heat

$$\begin{aligned} H &= h_a \rho C_p(\theta_0 - \theta_z)U_z \\ &= D_{0-z}\rho C_p(\theta_0 - \theta_z) \end{aligned} \tag{4}$$

for water vapour,

$$\begin{aligned} E &= d_a \rho(q_s - q_z)U_z \\ &= D_{0-z}\rho(q_0 - q_z) \end{aligned} \tag{5}$$

where θ q denote respectively potential temperature and specific humidity; C_d is the drag coefficient; h_a is the Stanton number; D_a is the Dalton number. The factor D_{0-z} is known as the integral-diffusivity or outer-diffusivity (BUDYKO 1956, BUDOGOVSKY 1964) and defined by

$$D_{0-z} = \frac{1}{\int_0^z \frac{dz}{K_H}} \quad \text{or} \quad \frac{1}{\int_0^z \frac{dz}{K_W}} \tag{6}$$

where K_H is the eddy diffusivity of heat and K_W is the eddy diffusivity of water vapour.

Furthermore we can see from equation (2) that D_{0-z}/U_z depends on a stability parameter. In fact a relationship between D_{0-z}/U_z and the stability ratio was shown by Russian workers (BUDOGOVSKY, 1964).

3. A comparison of H and E estimated by profile method with those obtained by integral diffusivity method

We will compare H and E estimated from profiles with those obtained by the integral diffusivity method. We adopt Monin-Obukhov's equation for profiles of wind and temperature in cases of near-neutral conditions and the Keyps equation for profiles in cases of unstable conditions. In Monin-Obukhov's model, stability parameter z/L is determined by the following relation:

$$z/L=f(z_0, B, \beta) \tag{7}$$

where

$$B=-\frac{g}{T_0}\frac{T(z_1)-T(z_3)}{U^2(z)} \tag{8}$$

and β is assumed as 3.25; z_0 is around 5 cm through all the runs. Then the friction velocity u_* can be expressed as

$$u_*=\frac{kU_z}{\ln\frac{z}{z_0}\left(1+\frac{\beta}{\ln\frac{z}{z_0}}\frac{z}{L}\right)} \tag{9}$$

and $K_M(\equiv K_H)$ at a height of z can also be expressed as

$$K_M=\frac{ku_*z}{1+\frac{z}{L}} \tag{10}$$

Finally we may obtained H by the following equation

$$H=-\frac{C_p\rho kU_2(T_3-T_1)}{\ln\frac{z_3}{z_1}\left(1+\beta\frac{z_3-z_1}{z\ln\frac{z_3}{z_1}}\frac{z_1}{L}\right)} \tag{11}$$

and E in a similar way.

In Keyps' model, we adopt $\sigma=13.0$ as shown by Kondo's paper (KONDO 1962). Either of the fluxes H and E is estimated by the series expansion of the solution of Keyps's equation (SYONO and HAMURO 1961; OKAMOTO 1963). We also calculate the Reynolds stress τ, the drag coefficient C_d, dimensional quantities T_* and E_*, the stability parameters R and z/L, at each of the heights 0.5, 0.75, 1, 1.5, 2, and 3 m. The computation for these quantities and parameters are carried out by IBM 7090.

We here use the following integral diffusivity D_{z_1-z} between z_1 and z, in place of the ordinary integral diffusivity D_{0-z}:

$$D_{z_1-z}=\frac{1}{\int_{z_1}^{z}\frac{dz}{K_H}}=\frac{1}{\int_{z_1}^{z}\frac{dz}{K_W}} \tag{12}$$

The above-mentioned D_{z_1-z} is estimated by the graphical integration after ploting $1/K_H$ against z on section paper; $D_{0.5-2}$, $D_{0.5-3}$ and D_{1-3} are computed. Heat and water vapour fluxes, H and E, estimated by the profile method (averaged for six values of six sublayers from 0.5 to 3 m high) are compared with $D_{z_1-z}(T_{z_1}-T_z)$ and $D_{z_1-z}(q_{z_1}-q_z)$. The results are shown in Fig. 1. A straight line has been drawn by eye so that it will best fit the black marks which were estimated by the use of Keyps' model.

The ratio of H to $D_{z_1-z}(T_{z_1}-T_z)$ is 4.17×10^{-4} cal cm^{-3} °C^{-1}, which is about 1.44 times larger than $c_p\rho=2.89$ cal cm^{-3} °C^{-1} at 20°C.

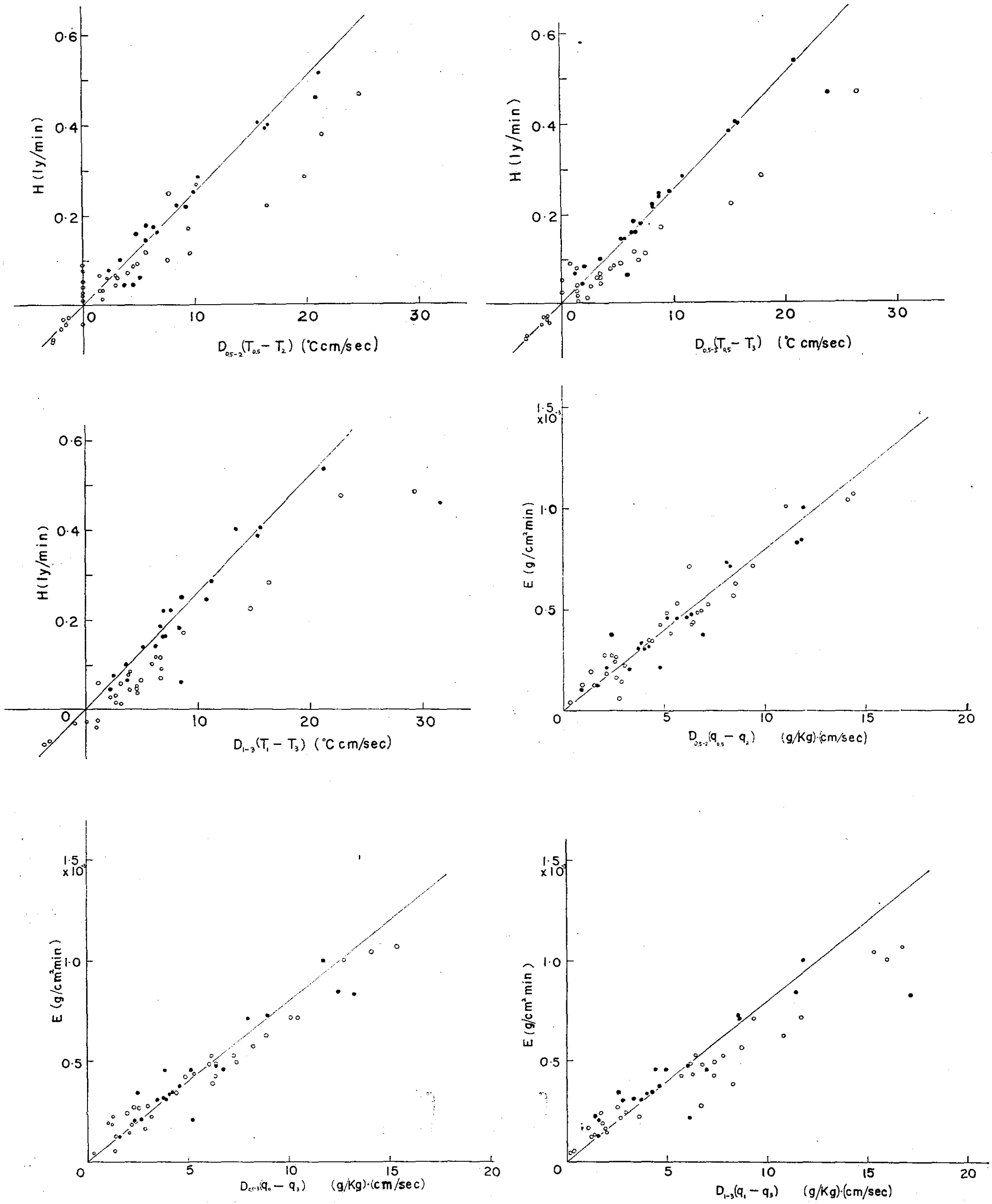

Fig. 1. Comparison of the profile method and the integral diffusivity method on the estimation of H and E. Black mark denotes values estimated by the use of Keyps' model and white mark Monin-Obukhov's model.

Similary, the ratio of E to $D_{z_1-z}(q_{z_1}-q_z)$ is 1.34×10^{-3}g cm^{-3}, which is about 1.13 times larger than $\rho=1.19 \times 10^{-3}$g cm^{-3} at 20°C. Much scattering of points in figures of H as compared with those of E reflect S wider variation of $c_p\rho$ than the small change of the air density ρ.

4. Comparison of D_{0-z} and D_{z_1-z}

The discrepancy between H estimated by the profile method and $c_p\rho\ D_{z_1-z}(T_{z_1}-T_z)$ is found to be 1.44. This is quite possible because the estimation of H is obtained, not by $c_p\rho\ D_{0-z}(T_0-T_z)$, but by $c_p\rho\ D_{z_1-z}(T_{z_1}-T_z)$.

Therefore an empirical relationship is sought between D_{0-z} and D_{z_1-z} as shown in Fig. 2, where the best fitted straight lines are drawn for the four cases, z_1=0.5 and 1 m; z=2 and 3 m.

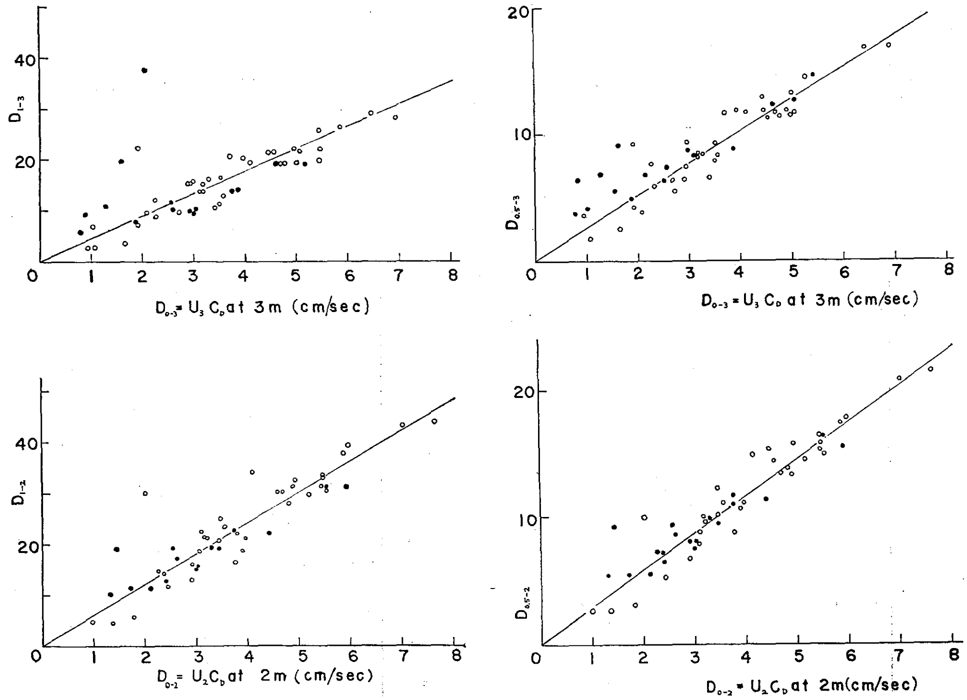

Fig. 2. The empirical relationship between integral diffusivity D_{0-z} and modified integral diffusivity D_{z_1-z}.

The integral diffusivity D_{0-z} is evaluated by equations (4) and (5). From Fig. 2, we obtain multipliers of D_{0-z} for each D_{z_1-z} as follows:

$$D_{0.5-1}=4.6D_{0-1} \qquad D_{1-2}=6.0D_{0-2}$$
$$D_{0.5-2}=2.9D_{0-1} \qquad D_{1-3}=4.5D_{0-3}$$
$$D_{0.5-3}=2.5D_{0-3}$$

Differences of magnitude of multipliers depend on the wind profile; this could be expected from the following deduction. The notation of H is differently adopted for D_{0-z} and D_{z_1-z}, that is,

$$H_{0-z}=C_p\rho D_{0-z}(T_z-T_0)$$
$$H_{z_1-z}=C_p\rho D_{z_1-z}(T_z-T_{z_1})$$

Assuming $H_{0-z}=H_{z_1-z}$, $u_*(z)=u_*(z_1)$ and $T_*(z)=T_*(z_1)$, we obtain

$$D_{z_1-z}=D_{0-z}\Big/\left(\frac{T_{z_1}-T_z}{T_0-T_z}\right)=\frac{D_0-z}{1-\dfrac{T_0-T_{z_1}}{T_0-T_z}}$$

$$=\frac{D_{0-z}}{1-\dfrac{-T_*kU_2/u_*}{-T_*kU_{z_1}/u_*}}=\frac{D_{0-z}}{1-\dfrac{U_{z_1}}{U_z}} \tag{13}$$

Therefore, the ratio of D_{z_1-z}/D_{0-z} depends on a factor $(1-U_{z_1}/U_z)$ for the fixed heights z_1 and z. When z_1 and z approach the same point, the ratio becomes larger; we can see this tendency in our cases.

An approximate linearity between D_{z_1-z} and D_{0-z} for the fixed heights z_1 and z as shown in Fig. 2 suggests that the variation of U_{z_1}/U_z is rather small except for lower values of D_{0-z} than 3 cm/sec although the ratio U_{z_1}/U_z should depend on the stability.

5. H_{0-z} and H_{z_1-z}; E_{0-z} and E_{z_1-z}

The assumption of $H_{0-z}=H_{z_1-z}$ can be checked as follows. The following relationship may hold if $H_{0-z}\neq H_{z_1-z}$

$$H_{0-z}=\frac{T_0-T_z}{T_{z_1}-T_z}\,\frac{D_{0-z}}{D_{z_1-z}}\,H_{z_1-z}$$

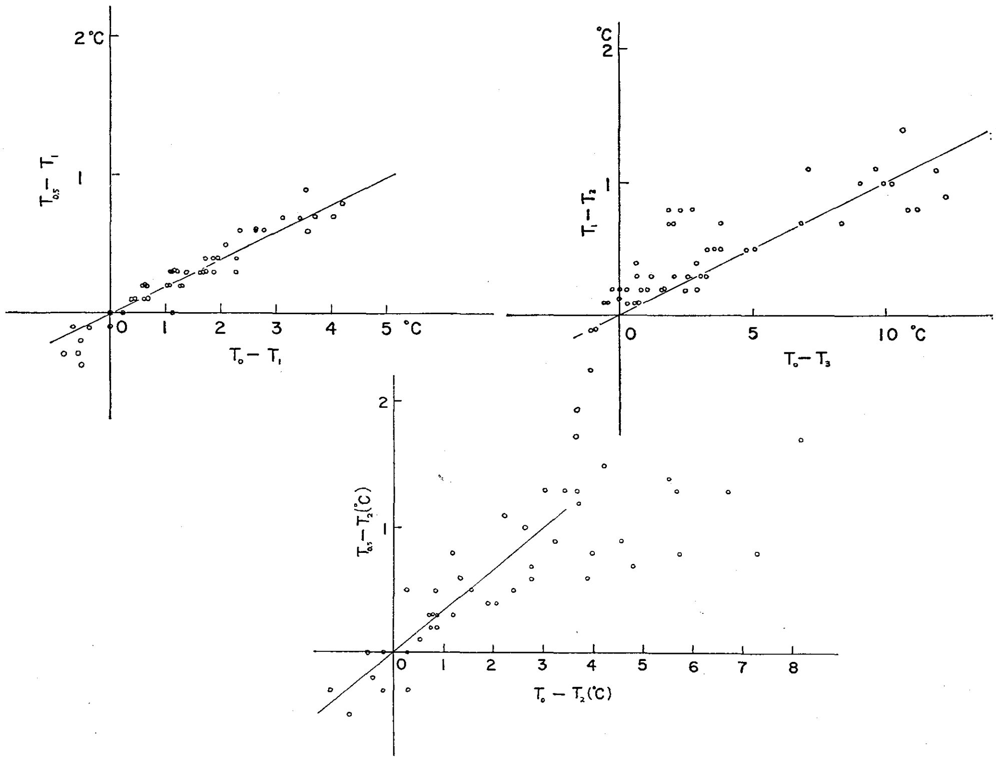

Fig. 3. The empirical relationship between T_0-T_z and $T_{z_1}-T_z$ for z_1 =0.5, 1 m and z=1, 2, 3 m.

Then the ratios of H_{0-z} to H_{z_1-z} are estimated by observational values of $T_0-T_z/(T_{z_1}-T_z)$ as shown in Fig. 3 for three cases among the combinations of z_1=0.5, 1 m and z=1, 2 and 3 m.

Observational values of $(T_0-T_z)/(T_{z_1}-T_z)$ are shown in Table 1.

Table 1. Observational values of $(T_0-T_z)/(T_{z_1}-T_z)$

$z(m)$ \ $z_1(m)$	0.5	1
1	5	/
2	3	9
3	2.5—5.0	10

With these values as well as the ratios of D_{0-z}/D_{z_1-z}, we obtain the ratios of H_{0-z} to H_{z_1-z} as shown in Table 2.

Table 2. Ratios H_{0-z}/H_{z_1-z}

$z(m)$ \ $z_1(m)$	0.5	1
1	1.09	/
2	1.03	1.50
3	1∼2	2.17

Table 2 shows a reasonable agreement of H_{0-z} and H_{z_1-z} for z=1, 2 m and for z_1=0.5 m,

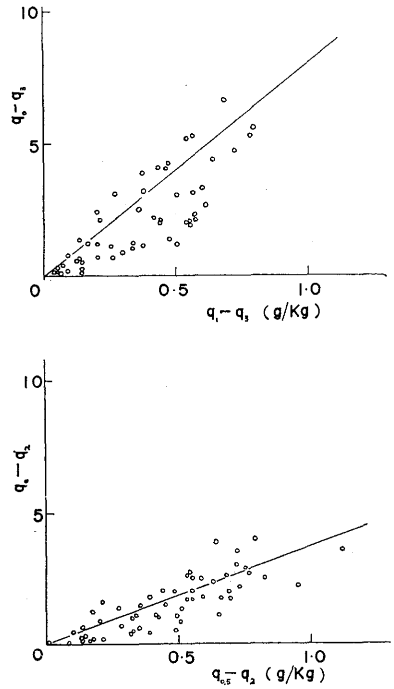

Fig. 4. q_0-q_z against $q_{z_1}-q_z$ for the two cases among the combinations of z=2, 3 m and z_1=0.5, 1 m.

Uncertain values of the ratio $H_{0-3}/H_{0.5-3}$ reflect scattered point of T_o-T_3, against $T_{0.5}-T_3$ in Fig. 3.
Heat fluxes, H_{1-2} and H_{1-3}, significantly differ from H_{0-2} and H_{0-3}, respectively. This difference shows that H_{1-2} and H_{1-3} estimated by D_{1-2} and D_{1-3}, respectively, give less values.

The same procedure is applied to obtain the ratio E_{0-z}/E_{z_1-z}. The empirical relationship between q_0-q_z and $q_{z_1}-q_z$ for two cases among the combinations of z=2, 3 m and z_1=0.5, 1 m are shown in Fig. 4
The ratios of q_0-q_z to $q_{z_1}-q_z$ are given in table 3.

Table 3. Ratios $(q_0-q_z)/(q_{z_1}-q_z)$

$z(m)$ \ $z_1(m)$	0.5	1
2	3.7	8.0
3	4.0	8.0

We similarly obtain ratios of E_{0-z} to E_{z_1-z} as shown in table 4.

Table 4. Ratio E_{0-z}/E_{z_1-z}

$z(m)$ \ $z_1(m)$	0.5	1
2	1.22	1.33
3	1.60	1.82

These large values of ratios E_{0-z}/E_{z_1-z} are due to much difference between q_o-q_z and $q_{z_1}-q_z$, which seems to reflect the accuracy of the measurement of water vapour or the structure of the transport of water vapour on a slightly rough surface.

6. Dependence of D_{z_1-z}/U_z on the stability

From equations (4) and (5), we obtain $D_{0-z}/U_z=h_a=d_a=f(z/L)$, assuming $K_H=K_M=K_W$. As the ratio D_{0-z}/D_{z_1-z} is rather constant for the fixed heights z and z_1 as shown in Fig. 2, D_{z_1-z}/U_z should depend on the stability parameter. We have here adopted the stability ratio as a stability parameter for practical convenience. The relationship between D_{1-3}/U_2 and $(T_1-T_3)/U_2^2$ is shown in Fig. 5, where the values obtained by Keyps' model are not sensitive to the stability in slightly unstable conditions.
Empirical formulae are obtained as follows:

$$\frac{D_{0.5-2}}{U_1}=4.7\left(1+3.40\frac{T_{0.5}-T_2}{U_1^2}\right) \tag{14}$$

$$\frac{D_{1-3}}{U_2}=4.7\left(1+6.38\frac{T_1-T_4}{U_2^2}\right) \tag{15}$$

or

$$\frac{D_{1-3}}{U_2}=4.7\left(1+7.87\frac{T_1-T_3}{U_2^2}\right)$$

$$\text{for} \quad \frac{T_1-T_3}{U_2^2} \leqq 0.2 \tag{16}$$

$$=4.7 \exp\left(3.53\frac{T_1-T_3}{U_2^2}\right)$$

$$\text{for} \quad \frac{T_1-T_3}{U_2^2} > 0.2$$

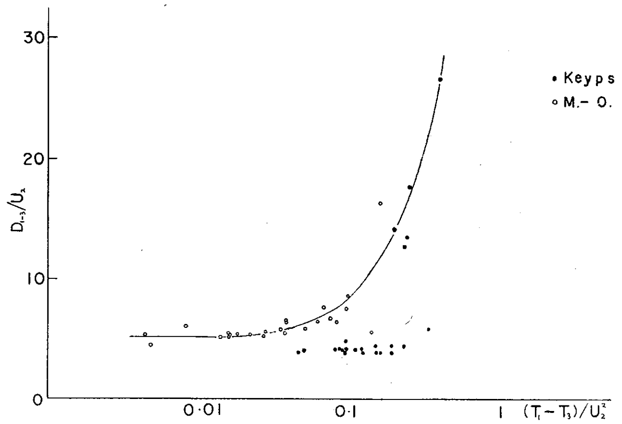

Fig. 5. The relationship between D_{1-3}/U_2 and stability ratio $(T_1-T_3)/U_2^2$.

7. An empirical bulk equation

We finally obtain H_{0-2}, H_{0-3} and also E_{0-2}, E_{0-3} through the measurement of the temperature at two heights and the wind speed at a height near the ground. These measurements are quite possible within the agro-meteorological network. The empirical bulk equations are as follows:

$$H_{0-2}=1.03H_{0.5-2}=1.03C_p\rho D_{0.5-2}(T_{0.5}-T_2)$$

$$=4.84C_p\rho U_1(T_{0.5}-T_2)\left(1+3.40\frac{T_{0.5}-T_2}{U_1^2}\right) \tag{17}$$

$$H_{0-3}=2.17H_{1-3}$$

$$=\begin{cases} 10.20C_p\rho U_2(T_1-T_3)\left(1+7.87\dfrac{T_1-T_3}{U_1^2}\right) \\ \qquad \text{for} \quad \dfrac{T_1-T_3}{U_2^2}\leqq 0.2 \\ 10.20C_p\rho U_2(T_1-T_3)\exp\left(3.53\dfrac{T_1-T_3}{U_1^2}\right) \end{cases} \tag{18}$$

and

$$E_{0-2}=1.22E_{0.5-2}$$

$$=5.75\rho U(q_{0.5}-q_2)\left(1+3.40\frac{T_{0.5}-T_2}{U_1^2}\right) \tag{19}$$

It may be seen from these empirical equations that estimation of H and E can largely change according to the ratios H_{0-z}/H_{z_1-z} and E_{0-z}/E_{z_1-z}, respectively. In particular, the ratios E_{0-z}/E_{z_1-z} may possibly change if subjected to a more refined measurement of water vapour. Therefore, for the water vapour flux, equations (19), (20) should be regarded as temporary, although we have done the most accurate measurement possible at the present stage. Another limitation is that all deductions are here derived under the assumption of $K_H=K_W=K_M$. Therefore, D_{z_1-z} for the heat flux and for the water vapour flux are assumed as the same. All the results obtained by the use of this same D_{z_1-z} may hold only in near-neutral conditions.

8. Conclusion

Empirical equations for the estimation of H and E near the surface of land based on the modified integral diffusivity D_{z_1-z} are presented here. In the process of the deduction of these empirical equations some critical work was done: comparisons of H and E estimated by the use of D_{z_1-z} with those obtained by profiles of wind speed, air temperature and water vapour; a comparison of ordinary integral diffusivity D_{0-z} with the modified integral diffusivity D_{z_1-z}, a comparison of H_{0-z} and E_{0-z} due to D_{0-z} wiith H_{z_1-z} and E_{z_1-z} due to D_{z_1-z}, the dependence of D_{z_1-z}/U_z on the stability ratio.

Empirical bulk equations for the estimation of H will be useful within the present agro-meteorological network of Japan with a little additional observation.

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References

Budogovsky, A. I., 1964: Испарение поивеснной влати. (in Japanese, translated by Z. Uchizima: The research committee on the technology of agricultural lands and forests. pp. 176)

Budyko, M. I., 1956: Тепловои валанс земнои поверхности. (in Japanese, translated by Z. Uchizima: The research committee on the temperature of water in rivers. pp. 181)

Deacon, E. L. and K. Webb, 1962: The sea. Chap II, 43–87, Interscience, New York.

Kondo, J., 1962: Observations on wind and temperatures profiles near the ground. Sci. Rep. Tohoku Univ. Series 5, Geophysics, **14**, 41–56.

Okamoto, M., 1963: A note on the wind and temperature profiles in the diabatic atmosphere near the ground. Geophys. Mag., **31**, 505–514.

Robinson, G. D., 1966: Another look at some problems of the air-sea interface. Quart. J. Roy. Met. Soc., **92**, 451–465.

Syono, S. and M. Hamuro, 1962: Notes on the wind-profile in the lower layer of a diabatic atmosphere. J. Met. Soc. Japan, **40**, 1–12.

バルク法による乱れの顕熱フラックスおよび水蒸気フラックスの推定

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温度，水蒸気に関し，地表面の値を用いないモデファイされた積分拡散係数を導入し，1966年千葉中央港埋立地で行なわれた野外観測（共同研究）の結果を用いてバルク法の検討を行なった。すなわち，傾度法によって求めた H および E と，バルク法によって求めたそれらとの比較を行ない，またモディファイされた積分拡散係数と安定度との関係を調べた。最後に，地表面の値を用いないで，H と E を評価する実験式を提案した。