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Wind Vane with Extremely Small Moment of Inertia and Its Possible Application

by

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Abstract

Wind vanes with reduced moment of inertia (one order lower than conventional) are tested in the wind tunnel. The results show a marked reduction in the overshoot amount, or a remarkable increase in the damping ratio. The nature of its effectiveness in suppressing the overshoot is different from that of an oil-damper. The shortcomings of the reduced rigidity can be remedied by utilizing a wind vane of ordinary design, but equipped with a rudder that acts upon the acceleration feedback of the vane oscillation.

1. Objective of the test

To prevent the overshoot of the wind vane or, what comes to the same thing, to increase the damping ratio, an oil-damper is an effective way as reported before [Sanuki, Kimura and Hayashi, 1965]. Another departure is the reduction of vane moment of inertia even if this means an increase in undamped natural frequency together with the damping ratio. However, if oscillation is well damped and overshoot is practically none, we shall suffer very little from the increased undamped natural frequency.

Therefore we started the manufacture of a vane model with extremely small moment of inertia, possibly one order smaller than conventional, to see its effect upon the vane oscillation. The objective of the test was thus first purely of theorectical nature.

2. Experimental set-up

To ensure ease of comparison, the shape and major dimensions of the model vane are the same as those of the former conventional one tested before (loc. cit.), and illustrated in Fig. 1. The only difference between the two is the thickness of the vane, which is 1.5mm for the present model, while it was 1mm for the former.

The reduction of moment of inertia is realized by employing very light balsa wood for the vane and its shaft instead of aluminium and brass. The result is a remarkable reduction in the moment of inertia (I) just as aimed at.

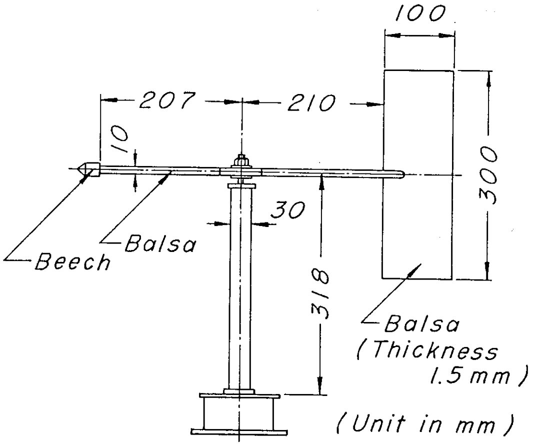

Fig. 1. Experimental set-up.

For the original aluminium vane with an oil-damper but without oil

$$I=18.1\times10^{-4}\ \mathrm{kgms}^2.$$

For the balsa vane with an oil-damper but without oil

$$I=1.71\times10^{-4}\ \mathrm{kgms}^2.$$

For the balsa vane without an oil-damper

$$I=1.32\times10^{-4}\ \mathrm{kgms}^2.$$

The reduction in moment of inertia inevitably caused the reduction in rigidity, and the balsa vanes are not suitable to the wind tunnel test above the wind speed of 10 m/s due to vane warping. Concerning this shortcoming, discussion will be made later.

Other details in experimenting are quite the same as described before (loc. cit).

3. Experimental result and discussion

In Fig. 2 the measured amplitude for wind speed $V=5$ m/s is plotted against time for both balsa vanes, while that for the aluminium vane is plotted in broken line. The tests are carried out also for $V=2.5$ m/s and 10 m/s and similar results are obtained.

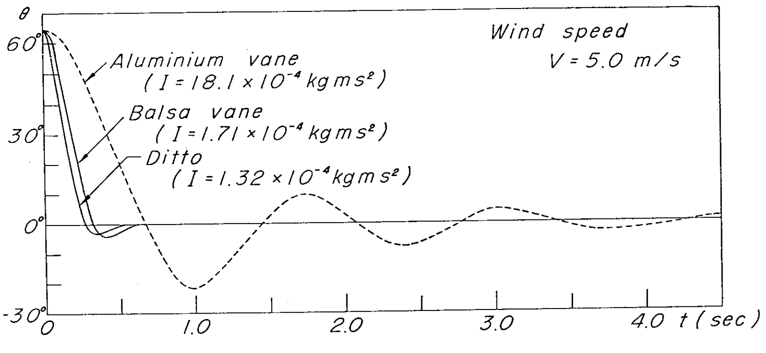

Fig. 2. Amplitude versus time of wind vanes with various moments of inertia (wind tunnel test).

In each case the amplitude curve is tightly compressed to the left, and practically no overshoot is observed. The tight compression is characteristic of the reduced moment of inertia, while a damper caused the curve to be rather loose and dull, overshoot being prevented equally in both cases.

Quantitatively expressed, for reduced moment of inertia, the damping ratio ζ increases according to the law (loc. cit.)

$$\zeta \propto \frac{1}{\sqrt{I}} \tag{1}$$

if other things* are equal.

The undamped natural frequency ω_n increases also according to the same law

$$\omega_n \propto \frac{1}{\sqrt{I}} \tag{2}$$

if other things** are equal.

The characteristic time, or the time required to reduce the initial envelope curve ordinate of amplitudes to $1/e$, $\tau = 1/\zeta\omega_n$, decreases in straight proportion to moment of inertia

$$\tau \propto I. \tag{3}$$

In case of an oil-damper ω_n remained unchanged and the damping ratio ζ increased linearly as the damper oil level increased, if other things were equal. Accordingly the characteristic time $\tau = 1/\zeta\omega_n$ decreased as the damping ratio increased, but not so rapidly as in the case of reduced moment of inertia.

These rules were based upon a simplified theory already referred to. In Fig. 3 the theory is compared with the experiment for the case of wind speed $V = 5.0$ m/s. The actual damping ratio ζ is smaller than that due to the inverse-root law, Equation (1), while the characteristic time $\tau = 1/\zeta\omega_n$ is larger than that given by the linear law, Equation (3). The discrepancies are due to the simplifications involved in the theory, which, however, certainly shows the general trends.

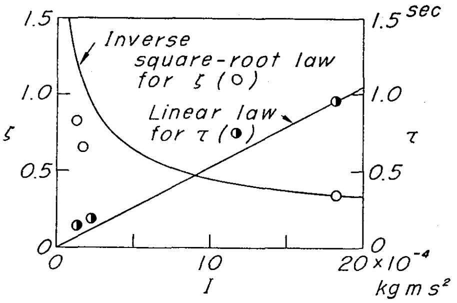

Fig. 3. Moment of inertia versus damping ratio ζ and characteristic time $\tau = 1/\zeta\omega_n$ ($V = 5.0$ m/s).

Also, according to the theory, ζ is independent of wind speed V, while τ changes inversely proportional to V, as ω_n increases proportionally to V. These postulations are well established in the experimental results though not illustrated here.

4. Acceleration-sensitive automatic rudder control of a wind vane

As pointed out before, the wind vane with reduced moment of inertia is not com-

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* The vane cross-sectional form, air density, vane area, and vane arm length are included herein.

** In this case the wind speed is included besides those mentioned in the foregoing foonote.

petent enough to withstand winds over $V=10$ m/s. The low rigidity of balsa caused a marked warp in the vane, which might yield altogether under much stronger winds.

A practical realization can be effected, however, by the use of a conventional, say, aluminium vane, but with a movable rudder which responds to the angular acceleration $\ddot{\theta}$. This requirement necessitates somewhat sophisticated devices such as the accelerometer and servomotor, even when they are utilized in aircraft automatic pilots.

Write the equation of motion without control as

$$I\ddot{\theta}+c\dot{\theta}+c'\theta=0 \tag{4}$$

where c and c' are constants. If a movable rudder is incorporated the equation will become

$$I\ddot{\theta}+c\dot{\theta}+c'\theta=M_\delta\delta \tag{5}$$

where M_δ stands for the aerodynamic moment due to a rudder angle δ. And as δ is acceleration-sensitive, it can be written

$$\delta=G\ddot{\theta} \tag{6}$$

where G is a proportionality coefficient, or, in the term of automatic control, a gain. Writing $M_\delta G=K$ we have

$$I\ddot{\theta}+c\dot{\theta}+c'\theta=K\ddot{\theta}. \tag{7}$$

The sign of K needs consideration. The effect of the rudder should prevent the growth of amplitude θ and thus the aerodynamic moment $K\ddot{\theta}$ should be negative for positive θ and positive for negative θ. However, as θ must always be of another sign than θ for a stable configuration, a positive K with a positive M_δ is required. Thus we get

$$(I-K)\ddot{\theta}+c\dot{\theta}+c'\theta=0. \tag{8}$$

In Equation (8), the value of $(I-K)$ can be made quite small by a suitable design, and an equivalent state of reduced moment of inertia can be realized.

5. Conclusions

1) The reduced moment of inertia of a wind vane causes an increase in damping ratio and, at the same time, an increase in undamped natural frequency.

2) As a result the overshoot is effectively suppressed and the vane movement or oscillation, if any, is completed in a shorter time than in the case of a vane with damper.

3) The reduced vane rigidity makes the actual use unpractical in strong winds, and a proposal is made to utilize a conventional vane with acceleration-sensitive rudder control. This needs some automatic devices such as the accelerometer and servomo-

tor, but, once realized, it will be extremely effective in preventing overshoot and terminating vane movement in a short time.

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Reference

M. Sanuki, S. Kimura and H. Hayashi, 1965: A Proposed wind vane with practically no overshoot. Pap. Met. Geophys., **16**, 84-89.

慣性能率を極度に小さくした風向計とその応用

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通常の風向計よりほぼ1桁だけ慣性能率の小さい風向計をバルサ材を使って作り，風洞実験してみた。その結果はゆきすぎ（オーバーシュート）量が激減し，同じことであるが減衰比（ζ）は激増した。減衰比を増すことはダンパーを装置しても実現できるが，この慣性能率を減じる方法は，そのほかに非減衰固有振動数（ω_n）を増すことが特徴である。したがって特性時間，すなわち，振幅の包絡線が初期値の $1/e$ に減じる時間（$\tau=1/\zeta\omega_n$）はさらに著じるしく低下できる。

この風向計は剛性が不足で，風速が大きいとき直接使用することはできない。しかし，角加速度を検知してフィードバックし，方向舵を操舵する装置が実現できれば，ゆきすぎ量を激減して，実用上効果のある風向計を得ることができることを示す点で意味がある。