

Static Stability Adjustment of an Adiabatic Quasi-nonlinear Model

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Abstract

An energy conserving atmospheric model is obtained by introducing the changes in the general field due to non-linear interaction of a disturbance which is governed by a linearized system of equations. Numerical integrations are made on meso-scale disturbances.

This model is characterized by the static stability change caused by non-linear heat transport, which in turn determines the life cycle in the growth of disturbances.

1. Introduction

As is well known, the static stability parameter is kept constant for geostrophic models and the averaged temperature field consequently cannot be changed. The static stability is undoubtedly one of the most important parameters for the atmospheric processes both of large and small scales since it determines the growth of disturbances which has the primary importance either in the general circulation or the convection.

Generally speaking, the disturbance of instability increases its kinetic energy. Therefore the $\overline{\omega\alpha}$ term has essentially a negative value. It should be remarked that the negative value of $\overline{\omega\alpha}$ shows the vertical heat transport as well as the energy transformation from potential to kinetic energy. The vertical heat transport has an effect to stabilize the stratification. LORENZ (1957) proposed the notion of gross static stability and discussed the static stability change in relation to the energy change.

The static stability change, which has not been considered in quasigeostrophic models, has an important effect on the instability problems in the mechanism of the atmosphere.

2. An energy conserving quasi-nonlinear system

Let us consider two-dimensional motion in (x, p) plane in a hydrostatic atmosphere

$$\begin{aligned}u &= U(p, t) + u'(x, p, t), \\v &= v'(x, p, t),\end{aligned}$$

$$\begin{aligned}\omega &= \omega'(x, p, t), \\ \alpha &= \bar{\alpha}(y, p, t) + \alpha'(x, p, t),\end{aligned}$$

where u, v are the horizontal velocity components, ω the vertical p -velocity and α the specific volume respectively. U is the zonal mean flow and $\bar{\alpha}$ is the zonal mean specific volume which is proportional to the zonal mean temperature. The zonally averaged field is indicated either by capital letters or bars, and perturbation quantities are indicated by primes which we will be able to omit in the following discussions without confusion.

Linearizing the equation of continuity, the equation of motion and the adiabatic equation, we obtain

$$(1) \quad \frac{\partial u}{\partial x} + \frac{\partial \omega}{\partial p} = 0,$$

$$(2) \quad \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + \omega \frac{\partial U}{\partial p} - f v = - \frac{\partial \phi}{\partial x},$$

$$(3) \quad \frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + f u = 0,$$

$$(4) \quad \frac{\partial \alpha}{\partial t} + U \frac{\partial \alpha}{\partial x} + v \frac{\partial \alpha}{\partial y} + \bar{\sigma} \omega = 0,$$

$$(5) \quad \alpha = - \frac{\partial \phi}{\partial p},$$

where ϕ is the geopotential and $\bar{\sigma}$ the static stability given by

$$\bar{\sigma} = \alpha \frac{\partial \ln \theta}{\partial p} = p^{\kappa-1} \frac{\partial}{\partial p} (p^{1-\kappa} \bar{\alpha}).$$

θ is the potential temperature and $\kappa = (R/c_p) = 0.286$. On the other hand, we have the zonal mean equations

$$(6) \quad \frac{\partial U}{\partial t} = - \frac{\partial}{\partial p} \bar{u} \omega,$$

$$(7) \quad \frac{\partial \bar{\alpha}}{\partial t} = - \frac{\partial}{\partial p} \bar{\omega} \bar{\alpha} - \frac{1-\kappa}{p} \bar{\omega} \bar{\alpha} = - p^{\kappa-1} \frac{\partial}{\partial p} (p^{1-\kappa} \bar{\omega} \bar{\alpha}),$$

showing that the vertical flux divergence of momentum and heat has an effect to change the zonal flow and the temperature field respectively. The temperature change is related to the static stability change since we have from eq. (7)

$$(7)' \quad \frac{\partial \bar{\sigma}}{\partial t} = - p^{\kappa-1} \frac{\partial^2}{\partial p^2} (p^{1-\kappa} \bar{\omega} \bar{\alpha}),$$

which shows that the static stability does change as a consequence of vertical heat transfer. These equations are not considered in the ordinary linearized models, since the system of equations (1)~(5) makes a complete system. However, by incorporating the nonlinear equations (6) and (7) to the linear system we can take into con-

sideration the important role of baroclinic instability and/or static stability in a simplified manner. This system is to be called "quasi-nonlinear model" because only the nonlinear effect on the general flow is taken into consideration among the nonlinear interactions of disturbances. PHILLIPS (1954) discussed the general circulation by employing a similar technique without feedback mechanism.

It can easily be shown that the equations (1)~(5) together with (6) and (7) construct an energy conserving system. Assuming that ω vanishes at the top and the bottom of the atmosphere ($\omega=0$ at $p=p_0$ and $p=0$), we obtain

$$(8) \quad \frac{\partial K'}{\partial t} = - \int_0^{p_0} u\omega \frac{\partial U}{\partial p} dp - \int_0^{p_0} \overline{\omega\alpha} dp,$$

from linearized perturbation equation of motion, and

$$(9) \quad \frac{\partial \bar{K}}{\partial t} = \int_0^{p_0} u\omega \frac{\partial U}{\partial p} dp,$$

$$(10) \quad \frac{\partial \bar{T}}{\partial t} = - \frac{p}{R} \overline{(\omega\alpha)}_p - \frac{1-\kappa}{R} \overline{\omega\alpha},$$

from zonal mean equations. \bar{K} and K' are the total kinetic energy of mean flow and perturbation. Combining (8), (9) and (10) we finally obtain

$$(11) \quad \frac{\partial}{\partial t} \left(\bar{K} + K' + \int_0^{p_0} c_p \bar{T} dp \right) = 0,$$

which shows that the summation of kinetic energy and total potential energy conserves.

3. Numerical integrations

The system of equations (1)~(7)' can be solved numerically under a given boundary condition and initial condition. Since the system is essentially linear except that the static stability and the general flow can change under the nonlinear interaction of a linear perturbation, a single harmonic wave disturbance with the wave length of L is considered in a shear flow without losing generality.

Let us consider that an unbalanced pressure disturbance is given initially in a slightly unstable atmosphere. Fig. 1 shows the initial stratification to be used for integrations. The integration was performed for various wave lengths ranging from 25 km to 400 km by means of 3-layer representation.

In Fig. 2 are shown the time changes of amplitude of vertical velocity to indicate the growth of disturbances. It is clearly seen that the growth of each disturbance has a life time which becomes longer if the wave length is longer and that the maximum growth becomes smaller for longer wave disturbances. The life times shown in Fig. 2 seem to coincide very well with the experimental facts observed in the meso-scale disturbances. SENSHU (1961) shows in his study on a meso-scale disturbance observed over Western Japan that the life time of the disturbance with a wave length of 100 km was about 8 hrs.

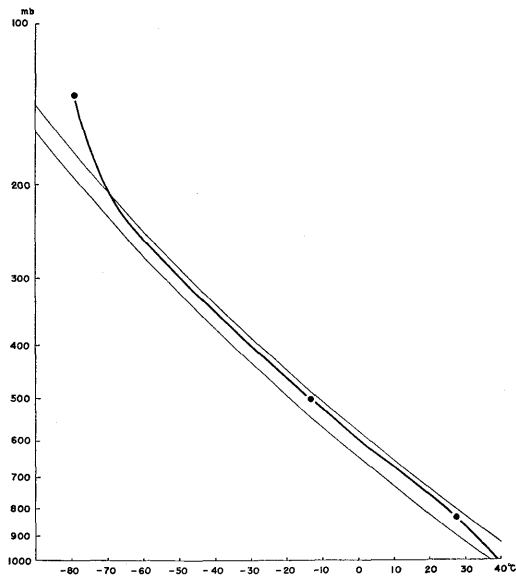


Fig. 1. The vertical temperature distribution which is employed for the initial condition of three-level model integration. Thinner lines are adiabats.

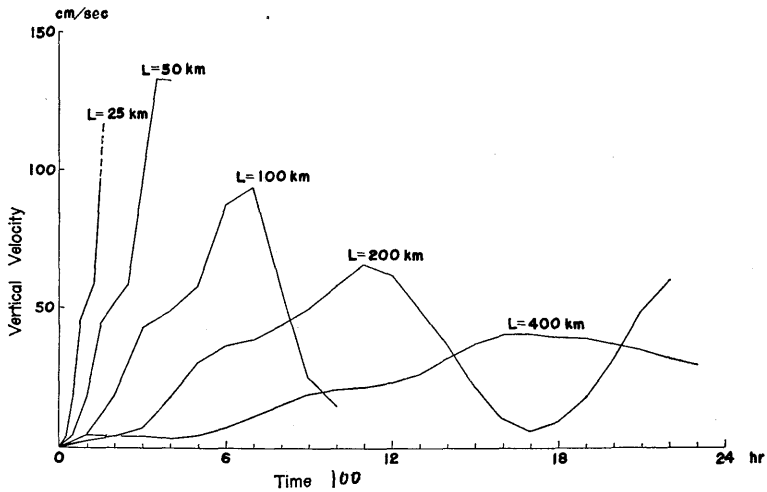


Fig. 2. The growth of disturbances with various wave lengths as a function of time. The ordinate is the amplitude of vertical velocity.

We can also find good agreements between the observation and model experiments in phase relationship and phase velocity. Fig. 3 gives an example of life history of pressure disturbance and vertical velocity. It is seen that the maximum upward velocity is found a quarter of wave length behind of the minimum pressure. If we assume that the precipitation is located within the area of upward motion, SENSU's (1961) and the authors' (1965) analysis reveal the same situation. Those circumstances undoubtedly are to be characterized as a kinematical structure of gravity waves in case of a slowly moving fluid whose substantial velocity is smaller than the phase velocity. Fig. 4. is quoted after SENSU (1961) for the sake of comparison.

It should be pointed out that the upward heat transport originating in a less stable atmosphere has an effect to eliminate the instability, while the downward heat transport in the stable stratification has an effect of destabilizing the layer. The stability change and the amplitude change of vertical motion as a measure of kinetic energy is given on Fig. 5 which shows a good coincidence between the two.

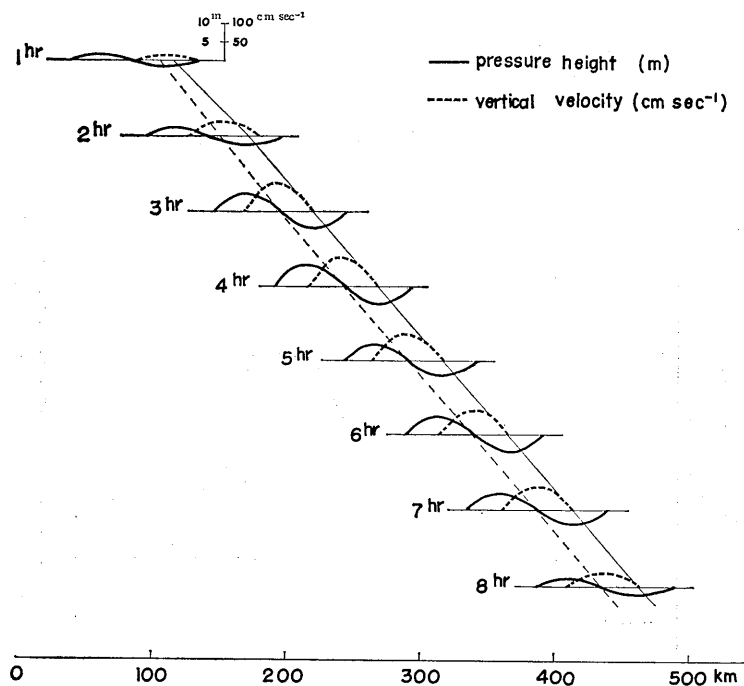


Fig. 3. The life cycle of a disturbance with a wave length of 100 km. The pressure height (full lines) and the vertical velocity (broken lines) distribution is given at every 1 hr interval.

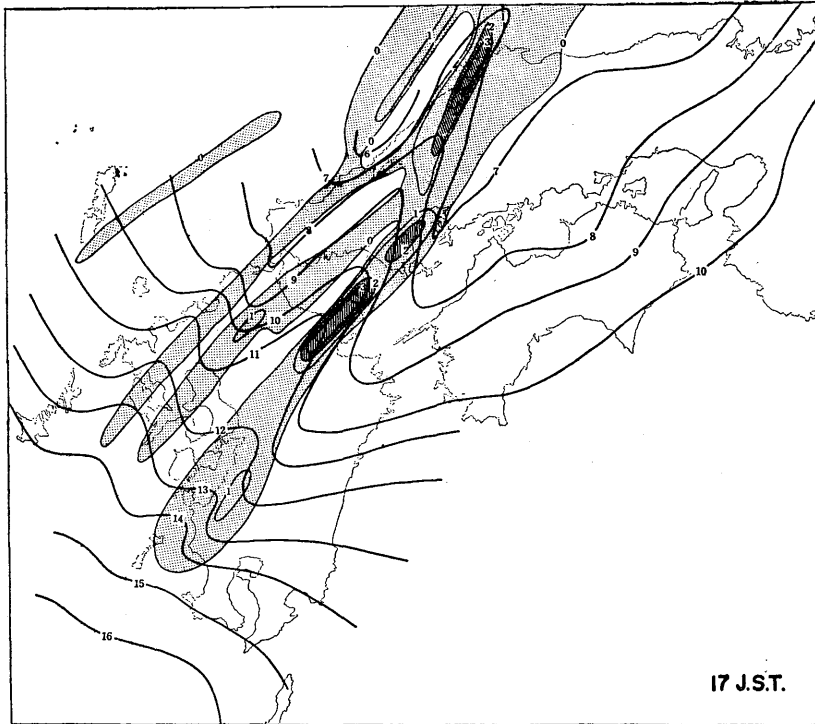


Fig. 4. An example of meso-scale disturbances. Isotherms are given by heavier lines and the rainfall area is hatched. (After Senshu, 1961)

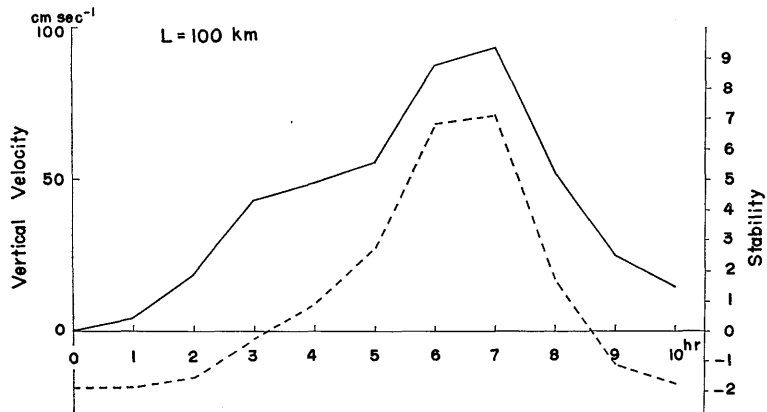


Fig. 5. The time change of the amplitude of vertical velocity (full line) and the static stability (broken line) for a disturbance with a wave length of 100 km.

4. Further simplification on the concept of quasi-nonlinear stability change

Let us further simplify our model to the two-level barotropic atmosphere as follows.

$$(12) \quad \dot{\omega} = -\frac{m^2}{n^2}\alpha,$$

$$(13) \quad \dot{\alpha} = -\sigma\omega,$$

$$(14) \quad \dot{\sigma} = n^2\bar{\alpha}\bar{\omega},$$

where $m=(2\pi/L)$ and $n=(1/\Delta p)$ represent the horizontal and vertical scale, i.e. wave length L and pressure depth of a layer, and the dots denote the operator $\partial/\partial t + U(\partial/\partial x)$. Eq. (12) is the equation of motion, eq. (13) the reversible adiabatic equation and (14) shows the static stability change due to quasi-nonlinear vertical heat transfer.

Denoting $C \equiv \bar{\omega}\bar{\alpha}$, $W \equiv \bar{\omega}^2/2$ and $A \equiv \bar{\alpha}^2/2$, we obtain

$$(15) \quad \dot{C} = -2\frac{m^2}{n^2}A - 2\sigma W,$$

$$(16) \quad \dot{W} = -\frac{m^2}{n^2}C,$$

$$(17) \quad \dot{A} = -\sigma C,$$

$$(18) \quad \dot{\sigma} = n^2 C.$$

As is easily seen, $(n^2/m^2)W$ is equivalent to the perturbation kinetic energy K' , and A/σ is the available potential energy. It is seen from eqs. (16) and (17) that C measures the energy conversion from kinetic to potential energy if the stratification of the mean field is stable ($\sigma < 0$).

It should be noted from eq. (15) that the upward heat transport is intensified as long as the available potential energy is larger than the kinetic energy ($A + \sigma K' > 0$). The increased upward heat transport ($C \ll 0$), on the other hand, makes the stratification much stabler and converts the available potential energy into kinetic energy. Therefore the direction of heat transfer could be altered after a certain degree of development is achieved.

We can easily obtain the equation of static stability after a simple elimination

$$(19) \quad \left(\frac{\ddot{\sigma}}{\dot{\sigma}}\right) = \frac{6m^2}{n^2}\dot{\sigma},$$

and similarly

$$(20) \quad \left(\frac{\ddot{K}}{\dot{K}}\right) = -6m^2\dot{K}.$$

The approximate solution of (20) is obtained by applying Taylor's series expansion with respect to time. Let us consider that a pressure disturbance is given initially in a calm and less stable atmosphere. Denoting the initial condition by subscript 0, we assume

$$\sigma_0 = K_0 = C_0 = 0.$$

The prime indicating the perturbation kinetic energy is omitted without confusion. Then we have

$$\begin{aligned} K_0 &= K_0^{\text{I}} = 0, \\ K_0^{\text{II}} &= 2 \frac{m^2}{n^2} A_0, \\ K_0^{\text{III}} &= K_0^{\text{IV}} = K_0^{\text{V}} = 0, \\ K_0^{\text{VI}} &= -18 m^2 (K_0^{\text{II}})^2 = -72 \frac{m^6}{n^4} A_0^2, \\ &\dots\dots\dots \end{aligned}$$

where the superscripts indicate the order of differentiation with respect to the time. Therefore we obtain

$$(21) \quad K \simeq \left(\frac{1}{2} K_0^{\text{II}} + \frac{1}{720} K_0^{\text{VI}} t^4 + \dots \right) t^2,$$

where t is the time. Let us define the life time T as a time in which the kinetic energy has approximately the initial value again. Eq. (21) gives

$$(22) \quad T^4 \simeq -360 \frac{K_0^{\text{II}}}{K_0^{\text{VI}}} = 10 \frac{n^2}{m^4 A_0}.$$

Similarly the value of the kinetic energy in its full development is approximated

$$(23) \quad K_{\text{max}} \simeq 1.2 \frac{\sqrt{A_0}}{n}.$$

Since the static stability changes proportionally to the kinetic energy change as is shown from eqs. (16) and (18) and is also seen in Fig. 5, we have

$$(23)' \quad \sigma_{\text{max}} = -n^2 K_{\text{max}} \simeq -1.2n \sqrt{A_0}.$$

We, then, finally obtain the approximate expression giving the life time

$$(24) \quad T \simeq \frac{L}{\pi \sqrt{K_{\text{max}}}},$$

by combining eqs. (22) and (23). It is shown that the life time of a disturbance is proportional to its scale.

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断熱準非線型モデルによる静的安定度調節

松本 誠 一

線型方程式に支配される擾乱の非線型干渉で基本場が変化することを導入して、エネルギー保存系の大気モデルを作ることができる。これを中規模擾乱にあてはめて数値積分を行なった。

このモデルの特長は非線型の垂直熱輸送により静力学的安定度が変化することであり、そのため擾乱の発達のライフサイクルが決められる。