

The Amount of Solar Radiation Falling on a Tilted Surface in Tokyo (Lat 35°41'N), Especially in Connexion with the Optimum Mounting Angle in the Outdoor Weathering Test

by

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Abstract

Solar and sky radiation falling on a tilted surface in Tokyo was calculated on the basis of sunshine recording data by use of the formulae proposed by PAGE. The results were compared with the observed global radiation and turned out to be in good agreement.

The efficiency of the radiation collector was discussed as a function of the sloping angle of the plate. It was concluded that the maximum efficiency is to be obtained by a plate of a slope lower by several degrees than that of the latitudinal angle itself. The order of magnitude of that difference proved to be 1% or so.

1. Introduction

The importance of the assessment of solar radiation is pointed out by many authors from the standpoint of solar energy utilization as in flat plate collectors and heating of buildings by solar radiation. The optimum mounting angle of the outdoor weathering test is another problem in this application especially concerning the effect of ultraviolet radiation.

So far, the measurement of solar radiation falling on a horizontal surface has been carried out by means of the Epply pyranometer, the Moll-type solarimeter and the Robitsch actinographs. In our country most observations were done by use of the Robitsch actinographs until the beginning of the IGY. The calibrations were chiefly done on the basis of the measurement of direct sun radiation through rather inadequate assumptions based on BERLAGE's formulae. On account of this unfortunate situation the reliability of data before the IGY is open to some criticism, which we would like to discuss in another paper.

Thanks to the effort of many people in charge of the solar radiation measurement at the Japan Meteorological Agency a new network based on the Eppley pyranometer has been started in the IGY. So the author was able to obtain sufficiently accurate data that have been calibrated following the international scale.

Concerning the assessment of solar and sky radiation falling on a tilted surface,

UCHIDA of Tokyo University has done a pioneering theoretical work and SAITO and others have recently presented comprehensive experimental and theoretical results. Especially the work of SAITO and others (1964) contains a continuous measurement of five years and also the assessment of solar radiation from the data of the sunshine record. Their work is still going on and is expected to yield fruitful results.

But considering from the present status of these problems the author thinks that the work of PAGE (1961) that was presented at the United Nations Conference at Rome on Applied Solar, Geothermal and Wind Energy contains the most comprehensive study on the assessment of solar radiation based on sufficiently numerous experimental data and also is very convenient because it gives the method of calculation of solar radiation falling on any desired tilted plane including the reasonable assessment of sky radiation.

So the author decided to make calculations based on PAGE's work.

2. The method of Page's calculation (1961)

The essentials of PAGE's method are composed of five steps, namely, 1. assessment of the daily total amount of solar and sky radiation falling on a horizontal surface, 2. that of diffuse sky radiation 3. separation of direct solar radiation, 4. that of direct solar radiation on a tilted surface, 5. that of diffuse sky radiation on a tilted surface.

The process of the 1st step is similar to that of Ångström's type based on the data of sunshine recorder.

Thus the daily total amount of solar and sky radiation Q is expressed as follows:

$$(1) \quad Q = Q_0 \left(a + b \frac{n}{N} \right).$$

Here Q_0 is the daily total amount of solar radiation falling on a horizontal surface outside the earth's atmosphere, n/N is the mean daily amount of bright sunshine hours divided by maximum possible ones.

This formula was already used by other authors but Page's distinction lies in the determination of the constants a and b . On the basis of tremendous data of solar radiation measurement and the simultaneous sunshine records at many stations from 40°S to 40°N he proposes that $a=0.23$ and $b=0.52$ are appropriate.

In the second step he proposes as the formula of diffuse sky radiation the following:

$$(2) \quad D_H = Q \left(c + d \frac{Q}{Q_0} \right).$$

This parabolical relationship between diffuse sky radiation D_H and global radiation Q is a characteristic of Page's theory and it seems adequate from many simultaneous observational results. He determined the constants as $C=1.00$ and $d = -1.13$.

Now the component of direct solar radiation is easily obtainable by simple subtraction of (2) from (1).

Then the process of converting the direct horizontal component to any tilted component is a logically simple one of straightforward trigonometric calculation, although, actually, it consists of very elaborate processes.

In the meantime Page did this calculation and made a table of conversion factors, so that the author could compute this amount from graphical interpolations.

Assessment of the contribution of diffuse sky radiation as well as that of ground reflexion is very difficult and remains imperfect because of such unknown factors as dust scattering and cloud reflexion.

At the present stage, however, it seems there is no other way than to assume a homogeneous isotropic sky radiation applying a simple geometrical consideration to both of them. The best possible way is to admit some factors for the sky brightness of the solar side to allow for the unbalance of intensity distribution due to the forward scattering of aerosol particles and also the forward reflexion by the cloud.

Thus the final formula can be written as follows:

$$(3) \quad \Sigma(I+D+r)_s = f_s \Sigma I_D \sin \theta + k \cos^2 \frac{\alpha}{2} \cdot D_H + \sin^2 \frac{\alpha}{2} \cdot r_H,$$

where

- $\Sigma(I+D+r)_s$: daily total of radiation falling on a tilted plane
- $\Sigma I_D \sin \theta$: daily total of direct sun radiation falling on a horizontal plane
- f_s : a factor to be multiplied to a tilted plane
- D_H : diffuse sky radiation falling on a horizontal plane
- k : a factor to be multiplied according as the sky radiation in this context is solar side or antisolar side, taking values between 1.2 and 0.8 rather arbitrarily
- r_H : a contribution due to ground reflexion and is expressed as $r_H = YQ$ where Y means albedo of the ground
- α : an angle of slope of the plane.

3. Assessment of global radiation at Tokyo (Lat 35°41') and the checking of Page's formula

Although the formula (1) is checked by PAGE on the basis of data for many places and periods, they were chiefly for stations in the Western world. So further study is necessary in order to establish its validity in the Asiatic area. And rather laborious statistical study will be required before we can arrive at a definite conclusion.

But, for the present, the principal objective of this paper is to investigate the influence of the angle of slope, so the author is content with a limited examination based on the IGY data at Tokyo only (J.M.A., 1957, 1958, 1962). Consequently the conclusion here arrived at should be considered as a qualitative one.

Table 1. Calculated and Observed Values of Global Radiation at Tokyo during IGY period. (cal/cm²/day), after eq. (1), and Ref. J.M.A. (1957, 1958).

'57 Time	July	Aug.	Sept.	Oct.	Nov.	Dec.
n/N	.41	.46	.21	.40	.53	.54
$Q(\text{cal})$	337	429	266	274	247	208
$Q(\text{obs})$	307	425	289	208	238	199
'58 Time	Jan.	Feb.	Mar.	Apr.	May	June
n/N	.62	.60	.43	.41	.47	.40
$Q(\text{cal})$	245	308	327	383	457	444
$Q(\text{obs})$	236	302	330	381	458	450
'58 Time	July	Aug.	Sept.	Oct.	Nov.	Dec.
n/N	.29	.39	.36	.34	.38	.62
$Q(\text{cal})$	376	392	329	255	208	224
$Q(\text{obs})$	362	385	285	246	201	206

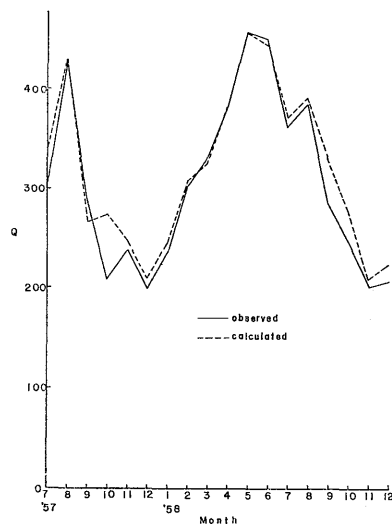


Fig. 1. Calculated and observed global radiation at Tokyo during the period of IGY (cal/cm²/day).

Table 1 and Fig. 1 show the data of sunshine record and the calculated global radiation as well as the observed one. As can be seen in Fig. 1, the general agreement could be said to be satisfactory both in general trend and in absolute values.

4. Radiation falling on a tilted plane and its efficiency

The slope of angle considered was 15°, 25°, 35° and 45° respectively. The percentage sunshine data were adopted from the Climatic Table published by the Japan Meteorological Agency. They are the mean monthly values covering 1931 through 1960.

As to the calculation of D_H , isotropic diffuse sky light was assumed without reference to the various tilting angles of the plate. This is not exactly true because it is obvious that the intensity of sky radiation becomes more and more intense as it becomes nearer to the sun both in clear and cloudy conditions. This problem is not finally settled, but Page discusses it considerably in detail, concluding that the difference is very small within the limited angles of slope now in discussion.

Concerning the effect of ground reflection the albedo was taken as 0.25, which is reasonable in the usual meteorological conditions of this district.

The original data and the details of the process of calculation not contained in the original paper of PAGE are tabulated in Table 2 and Table 3. The final results are given in Table 4. Here we are interested in the values of relative efficiency to those of the horizontal plane. They are given in the last column of Table 4 and shown in Fig. 2.

Table 2. Mean monthly amount of daily total flux of radiation outside the earth's atmosphere (Q_0), percentage of sunshine record (n/N), calculated global radiation (Q), calculated diffuse radiation (D_H), at Tokyo (Lat $35^\circ 41'$) (cal/cm²/day) (Calculated by eq. (1) and eq. (2)).

Month	1	2	3	4	5	6	7	8	9	10	11	12
Q_0	445	570	726	870	972	1010	990	912	783	622	484	408
n/N	60	34	47	46	45	34	41	49	37	39	47	56
Q	240	234	341	409	447	414	436	438	329	267	227	212
D_H	94	126	160	192	215	224	218	201	174	136	107	87
	146	108	181	217	232	190	218	237	155	131	120	125

Table 3. Ratio (f_s) of the mean monthly direct solar radiation falling on an inclined plane facing to equator as compared to that of horizontal plane, at Lat 35° (Calculated graphically from PAGE's Table).

Month	1	2	3	4	5	6	7	8	9	10	11	12
45°	1.96	1.58	1.25	0.97	0.80	0.73	0.75	0.88	1.11	1.44	1.81	2.05
35°	1.84	1.53	1.25	1.02	0.89	0.83	0.84	0.91	1.13	1.42	1.72	1.91
25°	1.67	1.45	1.21	1.06	0.94	0.91	0.92	1.00	1.13	1.34	1.58	1.71
15°	1.44	1.32	1.15	1.05	0.98	0.96	0.97	1.02	1.10	1.26	1.39	1.48

Table 4. Calculated shortwave radiation falling on an inclined plane at Lat 35° , by eq. (3) using mean sunshine record at Tokyo. (cal/cm²/day).

Month													Sum	Ratio
Angle of inclination	1	2	3	4	5	6	7	8	9	10	11	12		
45°	376	287	376	388	385	346	367	397	331	315	316	338	4221	1.057
35°	359	284	379	404	411	370	391	408	341	315	308	322	4292	1.074
25°	337	280	375	417	428	391	414	433	345	308	295	299	4322	1.082
15°	303	267	366	418	441	404	428	441	343	299	273	271	4254	1.065
0°	240	234	341	409	447	414	436	438	329	267	227	212	3993	1.000

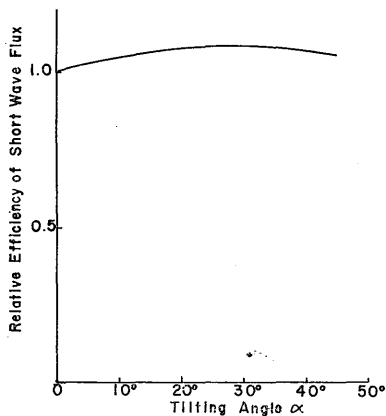


Fig. 2. Efficiency of flat plate solar energy collector with varying angle of slope (facing to equator).

Here it is noticeable that the maximum efficiency is attained at about $\alpha=25^\circ$, namely the slope lower by ten degrees than the latitude of Tokyo (L. $35^\circ 41'$).

Also in this figure we can notice that the order of magnitude of the gain of efficiency is about several per cent and the difference between $\alpha=25^\circ$ and 35° is about 1% or so.

In the mean time it is clear that $\alpha=45^\circ$ that was conventionally adopted in case of the weathering test of natural exposure is not adequate in our low latitudes.

5. Concluding remarks

The reason why the maximum efficiency appears at $\alpha=25^\circ$ instead of $\alpha=35^\circ$ in the curve of Fig. 2 is easy to see. The gain of solar radiation in the summer season will be large as compared with the loss in the winter season. This was already mentioned by G.C. NEWLAND and others (1963).

Now the details will be more clearly understood in Fig. 3 where the three different components of calculated values are drawn respectively when $\alpha=25^\circ$ and $\alpha=35^\circ$.

We can see the above-mentioned gain in the summer season in the curve of direct sun radiation. But it will be noticeable that the contribution of diffuse sky light is rather large and it also is accelerating that tendency. The contribution of reflected radiation from the ground is small and can be neglected.

Now we may safely conclude that both in the flat plate collector of solar energy and in case of the weathering test of natural exposure, the maximum efficiency will be obtained by the surface of a slope slightly lower, probably between 5° and 10° , than the angle of the latitude concerned.

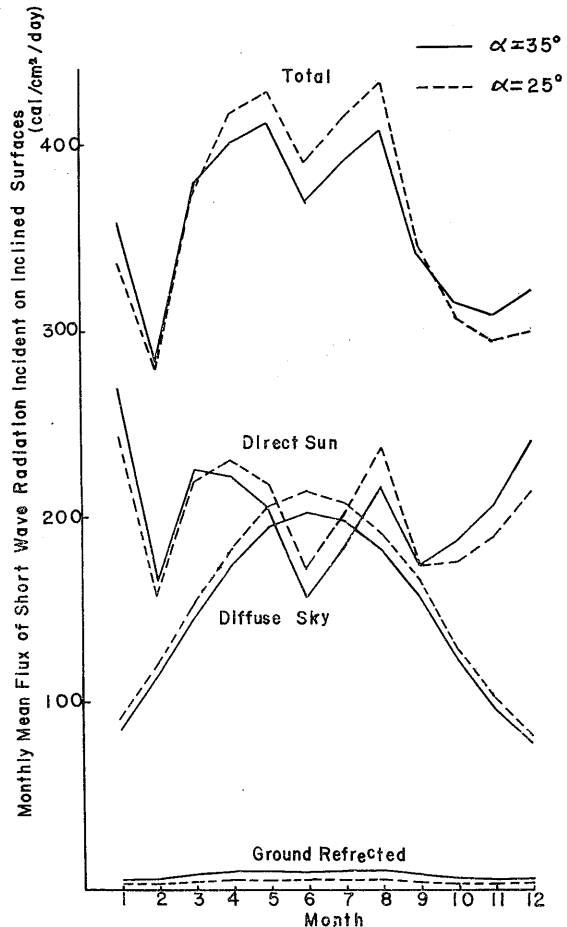


Fig. 3. Components of short wave radiation falling on varying inclined surfaces at Tokyo (Lat. $35^\circ 41'$).

α : angle of slope of the surface.

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東京における傾斜面への日射量

関 原 壘

東京における月別平均日射量1日積算値を PAGE の提出した経験式により日照率のデータを用いて計算した。水平面日射量の計算値は IGY の期間についての実測値と、非常によい一致を示し、この方法が有効であることを示している。

傾斜面については赤道方向にむかい 45° , 35° , 25° , 15° の各傾斜角の面への日射量を計算した。その結果日射量受熱の効率としては、緯度 35° の東京における最有効傾斜角は 25° 附近にあり、その効率の得は 1% 程度であることが判つた。

この原因は夏季における受熱量の得が冬季における損失を上廻っているためであるとして説明される。