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Eulerian Correlation of Temperature in the Atmosphere and Oceans

by

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Abstract

TOMODA's method for calculating the correlation coefficients $R(t)$ has been applied to temperature analysis in the atmosphere and oceans. The results show that the temperature fluctuations are caused by turbulence $m=4/3$, diurnal variation $m=2$, and some factor related to temperature durability $m=1$, being represented $R(t)\sim 1-(t/T_0)^m$. The maximum passage time of turbulence was $T_0=30$ hr.

Following INOUE and IMAI's result (1955), the 4/3 power law of the Eulerian correlation function of the temperature fluctuation due to the turbulence is to be applied for the regions of $t/T_0<0.1$ and of $1-R(t/T_0)<0.1$. Here the author presents the results of computation of the auto- and cross-correlation by TOMODA's method (1954) for various time scale in the atmosphere and oceans, assuming the sample distribution to be normal.

As the correlation functions are reduced to a formula such as $1-R(t/T_0)\sim(t/T_0)^m$, the results of computation of $R(t)$ for various time scales and at various locations are shown in the following figures and table.

From the results, the 4/3 power law will be supported by some of them, provided $T_0\leq 30$ hr. Namely there was not such a large turbulon that its passage time exceeded 30 hours. This large turbulon may be caused by a passage of trough and ridge in a westerly flow. In this case, curve No. 5, September 4th$\sim$5th, the tropical storm "Jane" passed 420 miles off the fixed point (39°N, 153°E), but it did not cause this turbulence. For the horizontal length of the storm $\Lambda_u=7°$ and its mean velocity $U=8°/24$ hr are estimated from the weather map, so its passage time is $T_u=21$ hr. By theoretical consideration, $T_u/T_0=2$ (INOUE, 1957). So $T_0=T_u/2=10.5$ hr. Thus the above value $T_0=30$ hr may be caused by the larger turbulence which may be concerned with the general circulation, as $T_*=192$ hr.

And as the averaging time $_*T$ for measurement would be long, such as the order of an hour, a day or a month, the value of m might become $m\rightarrow 1$. The auto-correlation function of the air temperature as $_*T=1$ day is empirically reduced to the relation $R(\tau)=e^{-\tau/\tau_0}$ (TAKAHASHI, 1951). The above relation is obtained from the former equation as follows:

$$R(\tau)=1-(\tau/\tau_0)^m$$
$$\doteqdot e^{-(\tau/\tau_0)^m},\quad 0\leq(\tau/\tau_0)^m<1$$
$$=e^{-\tau/\tau_0},\quad \text{as}\quad m=1,\quad 0\leq(\tau/\tau_0)<1.$$

INOUE and IMAI's theoretical result is

$$m=1\quad \text{for}\quad 0.1<t/T_0<0.5,$$
$$m=2/3\quad \text{for}\quad 0.5<t/T_0<1.$$

From the calculated results it seems to us that the temperature fluctuations of a large time scale discussed here, whose $m=1$, were not caused by the turbulence, but that they may be caused by some factor related to their durability or long periodicity.

Next when the averaging time T_* under analysis would be extended too long as compared to T_0 or ${}_*T$, the influence of seasonal variation would come in, even the diurnal periodic fluctuation will be analysed when there is no measurable turbulence. For example, the correlation function such as $1-R(t/T_0)=\{t/(T/4)\}^{5\cdot 7/3\sim 2}$ may be given by the periodic temperature variations whose period is T, namely $m=2$ means that this correlation function may be caused by diurnal variation and may be represented by a parabolic formula near $t=0$.

About these results at the fixed point, larger turbulons were seen in summer than in the other seasons when the value of m seems to take $m=1$ or $m=2$. And cross-correlations between the air and the water temperature are $R=0.3$ as $m=4/3$ and $R=0.7$ as $m=1$ or $m=2$.

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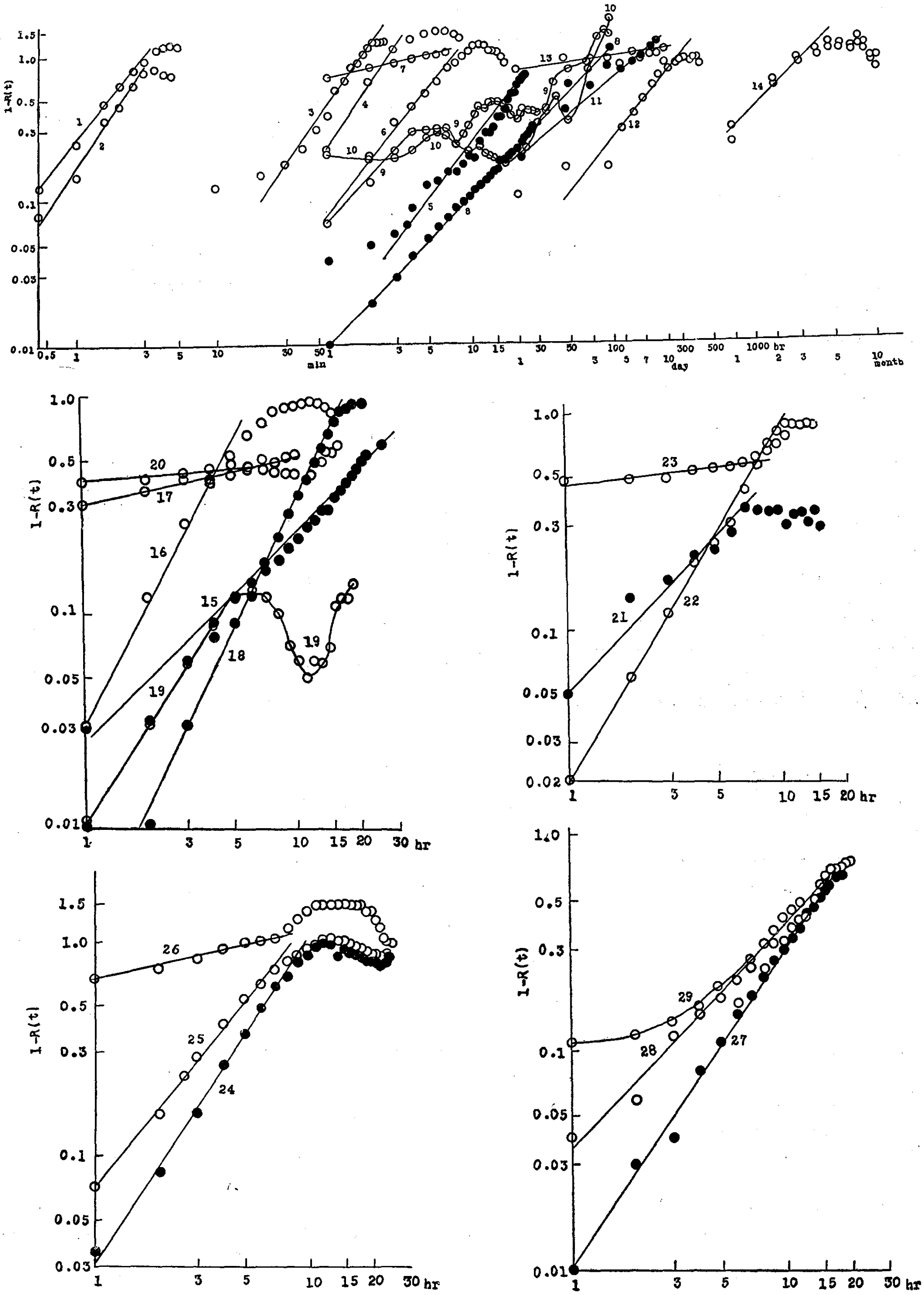

Fig. 1. Relation between $1-R(t)$ and time from auto-correlation of air and sea temperatures, and cross-correlation between them. The number attached to the line shows the curve number in Table 1.

Table 1. Auto- and cross-correlation coefficient in the atmosphere and oceans.

Curve no.	Material	Date	Location	Depth	$_*T$	T_*	$(1-R)\sim(t/T)^m$	Mean temp.°C
1	water	Aug. 10, 1939	off Osioro, Hokkaido	17.5 m	0.5 min	50 min	$(t/3\text{min})^{3.8/3}$	15.5
2	water	〃	〃	15 m	〃	60 min	$(t/3.3\text{min})^{4.2/3}$	17.0
3	water	Sep. 16~18, 1952	35°27.5'N 139°44.3'N	12.5 m	10 mm	54 hr	$(t/1.8\text{hr})^{4/3}$	22.8
4	water	Aug. 13~14, 1910	"Michael Sars" Station 115	300 m	1 hr	24 hr	$(t/2.8\text{hr})^{4.5/3}$	5.2
5	air	Sep. 1~8, 1950	39°00'N 153°00'N	0 m	1 hr	192 hr	$(t/30\text{hr})^{3.9/3}$	23.6
6	water	〃	〃	〃	〃	〃	$(t/7.0\text{hr})^{4.1/3}$	24.4
7	air-water	〃	〃	〃	〃	〃	$(t/4.4\text{hr})^{0.2}$ $R=0.35,\ t=0$	
8	air	Oct. 1~8, 1950	〃	〃	〃	〃	$(t/94\text{hr})^{3.0/3}$	17.4
9	water	〃	〃	〃	〃	〃	$t^{3.2/3}$	20.7
10	air-water	〃	〃	〃	〃	〃	——— $R=0.75,\ t=0$	
11	air	Aug.-Sep.-Oct., 1950	〃	〃	1 day	90 days	$(t/170\text{hr})^{2.1/3}$	23.1
12	water	〃	〃	〃	〃	〃	$(t/270\text{hr})^{3.6/3}$	24.4
13	air-water	〃	〃	〃	〃	〃	$(t/220\text{hr})^{0.1}$ $R=0.37,\ t=0$	
14	water	1940~55	Izuhara	〃	1 month	16 years	} $(t/2500\text{hr})^{2.8/3}$	
〃	water	1941~53	Aikawa	〃	〃	13 years		
15	air	Mar. 10~17, 1950	39°00'N 153°00'E	〃	1 hr	192 hr	$(t/40\text{hr})^{3/3}$	4.0
16	water	〃	〃	〃	〃	〃	$(t/7\text{hr})^{5.7/3}$	7.6
17	air-water	〃	〃	〃	〃	〃	$t^{0.29}$ $R=0.73,\ t=0$	
18	air	May 10~17, 1950	〃	〃	〃	〃	$(t/17\text{hr})^{6/3}$	14.6
19	water	〃	〃	〃	〃	〃	$(t/18\text{hr})^{4.8/3}$	14.8
20	air-water	〃	〃	〃	〃	〃	$t^{0.1}$ $R=0.60,\ t=0$	
21	air	July 10~17, 1950	〃	〃	〃	〃	$(t/18\text{hr})^{2.9/3}$	22.9
22	water	〃	〃	〃	〃	〃	$(t/11\text{hr})^{4.5/3}$	22.5
23	air-water	〃	〃	〃	〃	〃	$t^{0.13}$ $R=0.5,\ t=0$	
24	air	Sep. 15~22, 1950	〃	〃	〃	〃	$(t/10\text{hr})^{4.5/3}$	23.8
25	water	〃	〃	〃	〃	〃	$(t/8.3\text{hr})^{3.8/3}$	24.9
26	air-water	〃	〃	〃	〃	〃	$(t/6\text{hr})^{0.3}$ $R=0.38,\ t=0$	
27	air	Nov. 28~ Dec. 5, 1950	〃	〃	〃	〃	$(t/22\text{hr})^{4.5/3}$	14.0
28	water	〃	〃	〃	〃	〃	$(t/24\text{hr})^{3.1/3}$	14.9
29	air-water	〃	〃	〃	〃	〃	——— $R=0.9,\ t=0$	

$_*T$: averaging time for measurement, T_*: avaraging time under analysis.

気温及び水温のオイラー相関

南日　俊夫

友田の方法を用いて気温及び水温の自己相関ならびにそれらの間の相関を求めた。その結果 $T_0 \leq 30\,\mathrm{hr}$ で $^4/_3$ 乗則が成立つ事が示された。即ち通過時間が 30 hr を越える乱子は存在しなかつた。そしてこの大きな乱子は偏西風のトラフやリツヂの通過によるものであろう。測定の平均化時間が長くなる，即ち時間・日・月のオーダーになると $^4/_3 \rightarrow 1$ となる。このばあひ温度変化は乱流によるのではなく，温度の持続性とか長週期に関係した因子によるものと考えられる。又解析すべき資料の時間を乱子の通過時間や測定平均化時間に比べあまり長くとると，長週期変化の影響が入つて来て相関係数は時間の自乗に比例して来る。即ち計りうべき乱子が存在しない時は，日変化を解析していることになろう。例えば $1-R(t/T_0)=\{t/(\tau/4)\}^2$ の形で表わされる相関函数などは週期 T の温度変化によるものであろう。即ち $m=2$ と云う相関係数は温度の日変化により起るものであつて，その形は $t=0$ の近傍では抛物線で表わされる。北方定点では夏季に大きな乱子が見られ他の季節では $\mathrm{m}=1$ 或は$\mathrm{m}=2$ になりがちである。そして水温と気温の相関は $\mathrm{m}={}^4/_3$ の時 $R=0.3$，$\mathrm{m}=1$ 或は 2 の時 $R=0.7$ となる。一般に

$$R(\tau) \sim 1-(\tau/\tau_0)^m$$
$$\fallingdotseq e^{-\tau/\tau_0}, \qquad \mathrm{m}=1 \quad 0 \leq (\tau/\tau_0) < 1$$

これは気温について既に得られている経験則である。