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On the Structure of Ocean Currents (II)

— The Turbulent Fluctuations in a Tidal Current —

by

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Abstract

The component of turbulent velocity in the direction of the mean flow has been studied for the tidal current off Cape Kan'non. The periods of the turbulent fluctuations recorded varied from several seconds up to a few minutes and these fluctuations seem to be caused by turbulence. The ratio of mean amplitude of fluctuations to the mean current was greatest at the bottom and smallest at mid-depths. The calculated values of $R(t)$, the auto-correlation coefficients, were seldom in accord with the theoretical result as $1-R(t)\propto t^{2/3}$.

1. Introduction

Observations have previously been made of turbulent fluctuations in the speed and direction of the tidal current at a short distance from the coast of Cape Kan'non using the author's photoelectric current meter [1]. On the same station, the instrument used was the other type of the author's [2] on March 21 and 25, 1956. From the series of observations, it was possible to derive some values for the auto-correlation coefficients and to obtain interesting results about dependence of the fluctuation ratio u/U on the distance from the bottom.

2. Experimental results

The observations were made in Uraga strait on board the research ship "Asasio Maru". The station was shown in the previous paper [1], and the predicted tide at Mera is shown in Table 1.

Table 1. Predicted tide at Mera.

Date	High water				Low water			
	Time	Height	Time	Height	Time	Height	Time	Height
Mar. 21	0125	105	1055	110	0600	92	1855	28
25	0345	131	1550	145	0950	34	2215	24

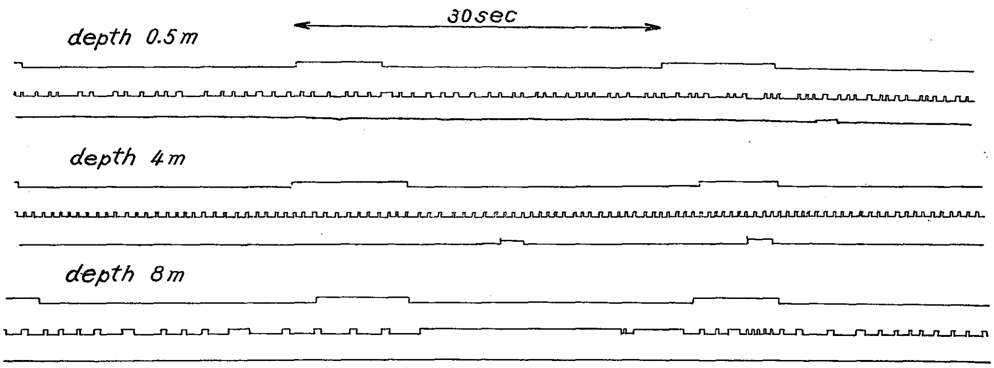

Fig. 1. Examples of recordings obtained at the surface, middle layer and the bottom, respectively.

Typical examples of the records on 21st are shown in Fig. 1. When water particles run 13 cm, the propeller revolves one cycle and the electric circuit is closed once. In the figure, the periodic or irregular fluctuations of about 2 or 3 sec at the surface, and the irregular ones at the bottom were seen. There was a reason to suspect that some of the fluctuations at the surface were due to ocean

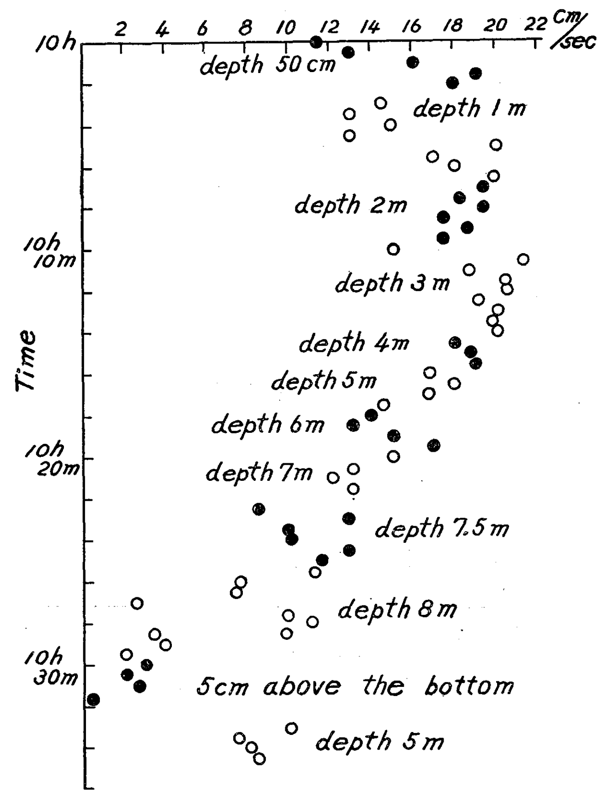

Fig. 2. Observed current at various depths.

waves and the fluctuations near the bottom were due to the turbulent flow by the boundary layer.

Now each record was divided into sections of 30 sec duration, and estimates made of the mean currents and the results were shown in Fig. 2. At 5 m layer, the speed decreased about 8 cm/sec after the interval of 17 min, so the tidal current might have decreased 0.47 cm/sec per one minute. From the results, the current profiles are shown in Fig. 3. The other result obtained on 25th at 10 m depth, the bottom layer, is shown in Fig. 4. In the observations only one anchor was dropped, and the ship was moved around as the current changed its direction. So the direction of the ship and the current formed an angle of about 180°.

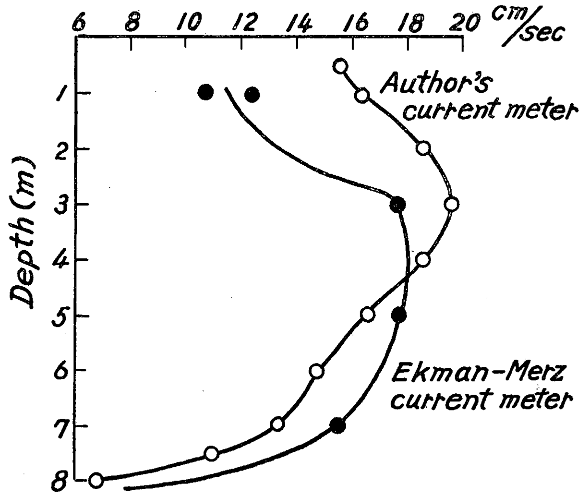

Fig. 3. The observed current profiles, their directions were 145°~165°.

3. Mean amplitude of fluctuations and the mean current

Most of the records on 21st covered intervals of approximately 3~5 min. Each record was divided into sections of 30 sec duration. The mean current U and the mean amplitude of fluctuation u were obtained by taking the average over all intervals. The ratio u/U was deduced and shown in Fig. 5. And the standard deviations of u are shown in Table 2.

Table 2. Variation of standard deviation of u with depth.

Depth (m)	0.5	1	2	3	4	5	(5)	6	7	7.5	8	5 cm above the bottom
Standard deviation	3.26	2.89	0.903	1.83	0.529	1.45	1.08	1.74	1.26	1.78	3.43	1.33
U (cm/sec)	15.5	16.3	18.5	19.5	18.6	16.5	8.5	14.7	13.3	11.0	6.8	1.9

The ratio u/U at the bottom layer between 8h 44 m and 9h 20 m on 25th, is also shown in Fig. 5. And its standard deviation of u is $s=3.47$, $U=10.4$ cm/sec.

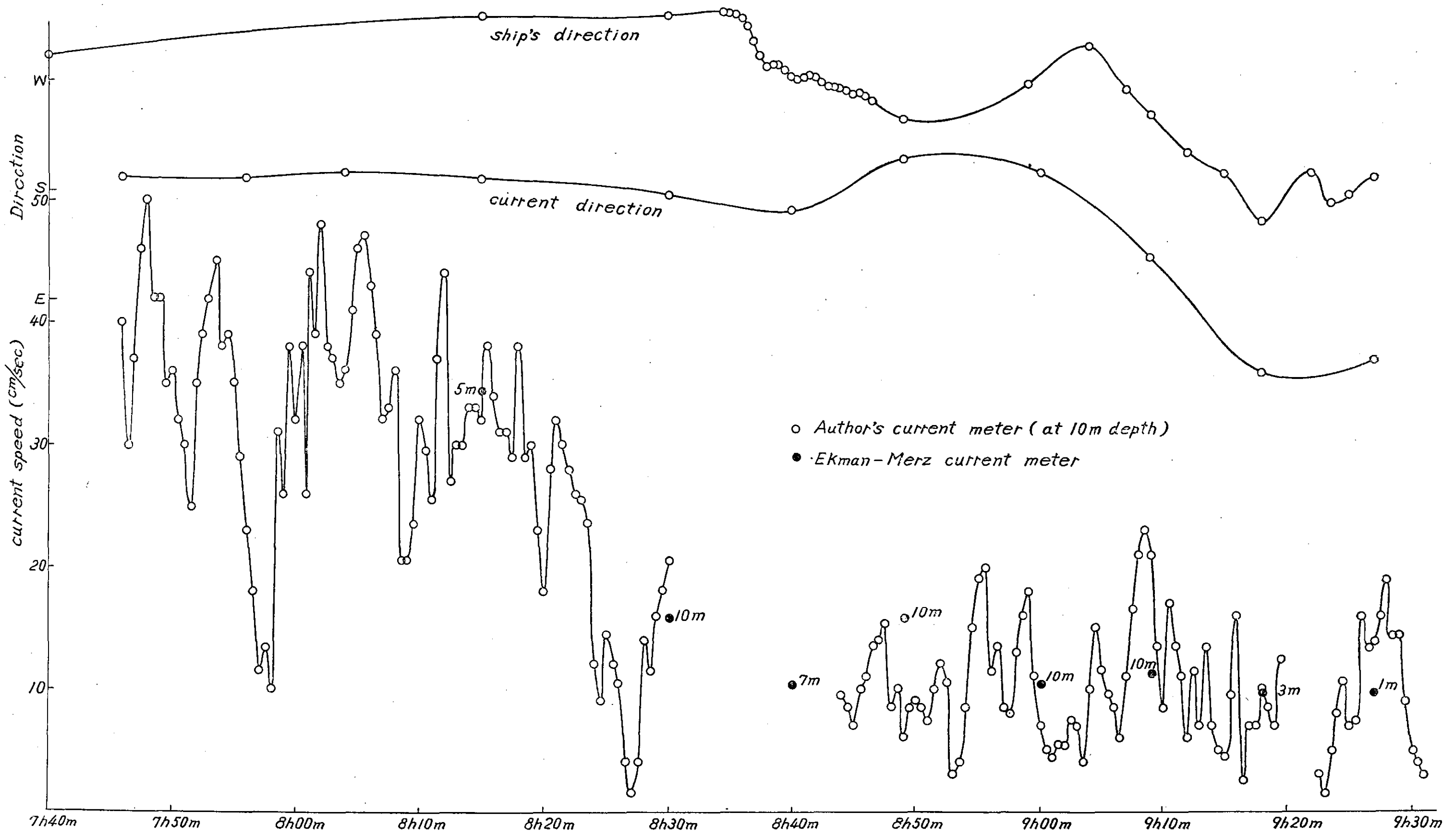

Fig. 4. Observed current at the layer of 10 m depth.

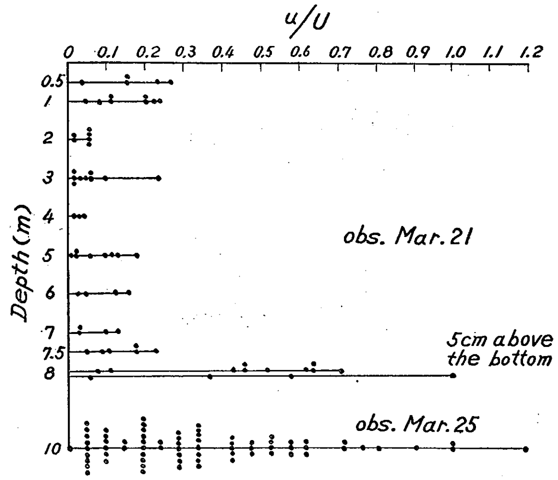

Fig. 5. Variation of current fluctuation ratio u/U with height above bottom and its frequency distribution.

From these, it comes out as a feature of the figure that near the surface the values of u/U are considerably less than those near the bottom, and the values at mid-depths are small. The distribution of values of u/U and also the standard deviation of u diminishes as the height above the bottom increases. Where high values do occur near the surface, there were periodic or irregular fluctuations, and they will be discussed later. The meteorological data are shown in the following table.

Table 3. Meteorological data.

Date	Air temp. °C	Weather	Beaufort wind scale	Sea	Swell
Mar. 21	9.2	fine	N. 1	N, 1	S, 1
Mar. 25	—	cloudy	SE. 2	SE, 1	SE, 1

4. Auto-correlation

The auto-correlation coefficient $R(t)$ for overlapping time interval or the averaging time for measurement $_*T$ is calculated. For the records on 21st, readings were repeated at regular intervals of 5 sec duration over the whole record. The results are shown in Fig. 6. The calculations were made near the surface (curve 6a), mid-depth (curve 6b) and near the bottom (curve 6c). In general such curves show a steady decrease in $R(t)$ with increasing values of time t. But the curve (6b) at 3 m depth shows that $R(t)$ undergoes oscillations of decreasing amplitude superimposed on a steady decay as t increases, having a period of about 20 sec. For the data on 25th, readings were repeated at intervals of 30 sec. The auto-correlation curve between 7h 46 m and 8h 30 m is shown as curve 7a in Fig. 7. Now the smaller fluctuation superimposed on the larger turbulence will be discussed. Eliminating the larger fluctuation, the auto-correlation curve 7b of the resultant smaller one is shown, and also the curve 7c between 8h 44m and 9h 20m is shown. The more minutely calculated value of the curve 7c is also shown in Fig. 6.

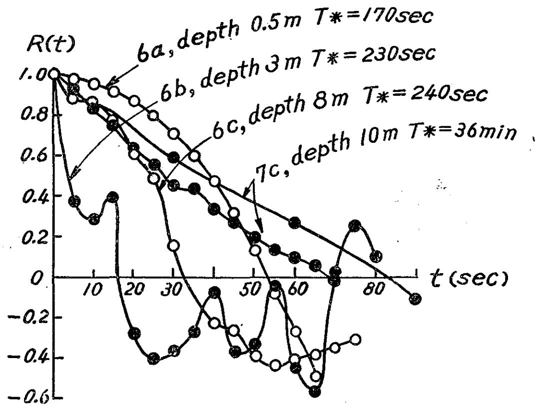

Fig. 6. Auto-correlation curves at various depths.

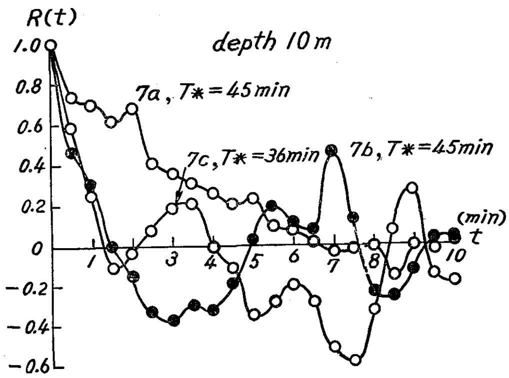

Fig. 7. Auto-correlation curves for longer time intervals.

The similarity theory for the inertia subrange of turbulence, or the range of intermediate turbulon gives the relations of $1-R(t)=(t/T_u)^m$. In general $m=2/3$, but it changes with the averaging time for measurement or the sensitivity of the instrument and the length of time under analysis, etc. [3] [4] [5]. The curves 6a and 6c show that the averaging time for measurement ${}_*T$ were too large to the passage time of the largest turbulon T_u, and so the correlation curve should be modified. To analyse these small turbulences, a more sensitive current meter should be used. The relation of $1-R(t)$ and time in these observations are illustrated in Fig. 8. For the curve 7a, the relation is expressed by $1-R(t)=(t/340)^{1.8/3}$.

When the averaging time T_* of observation or analysis would be too short to T_u, the characteristic quantity T_u should depend on T_* itself. It might be reasonable to choose T_* as about 10 times the anticipated T_u [5]. So the time $T_*=$ 2640 sec was a little short to about $T_u=340$ sec.

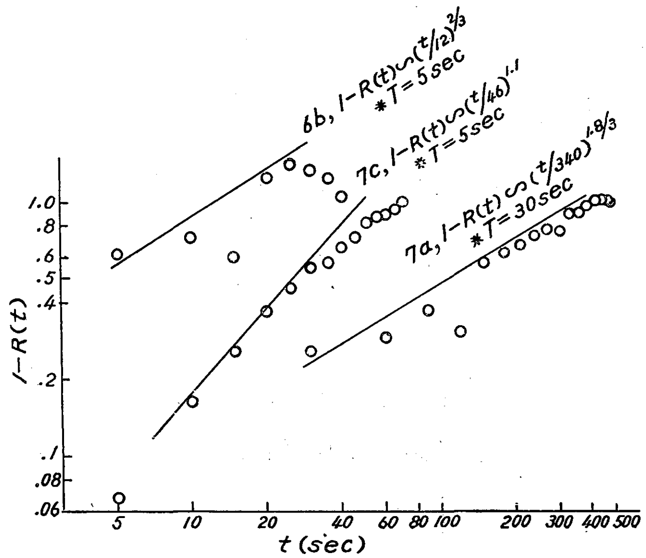

Fig. 8. Relation between $1-R(t)$ and time.

Next the relation $m=2/3$ is held provided that $R(t)>0.5$ and $<u^2>^{1/2}/U\ll 1$ [5]. About the curve 6b, $R(t)<0.5$ and $<u^2>^{1/2}/U=0.089$, so the relation can not be found exactly but it may be expressed by $1-R(t)=(t/12)^{2/3}$.

About the curve 7c, $<u^2>^{1/2}/U=0.33$ and T_u is about 46 sec. In this case, $1-R(t)\sim t^{1.1}$. It might also be that the correlation curve was changed from the original form by the low sensitivity of the instrument.

At any rate they may give the magnitude of the order as the mean length of eddy $l\sim U\int_0^{T_u}R(t)\,dt$, the diffusion coefficient $K\sim l<u^2>^{1/2}$ and the horizontal length of maximum turbulon $\Lambda_u=U\,T_u$ as follows:

Curve	Depth	U (cm/sec)	T_u (sec)	l (cm)	K (cg.s.)	Λ_u (cm)
6 b	3 m (mid-depth)	19.9	12	95.5	165	239
7 a	10 m (the bottom)	29.6	340	4026	—	10060

In conclusion, it was found that the calculated values of $R(t)$ were seldom in accord with the theoretical result, $R(t)=1-(t/T_u)^{2/3}$, because the conditions of observations seem to be under some limitations such as shortness of the averaging time T_* of observation or analysis and the low sensitivity of the instrument due to its momentum inertia, etc.

5. Periodic fluctuation

The shorter period fluctuation at the surface layer shown in Fig. 1 was due to surface waves, for its period of fluctuation coincided with the sea's, and its amplitude diminished and disappeared at the deeper layer. The longer-period fluctuations at 3 m and 8 m layers are discussed in the following.

If waves of a single period T are present among the fluctuations, it is known that their effect on the auto-correlation curve is to cause an undamped oscillation in $R(t)$ of period T. If the periods of the waves extend over a band, centered at T, the corresponding oscillation in $R(t)$ is damped. Now let the fluctuations be $u_t=u'_t+u''_t$. If $R(t)$ is defined by

$$R(t)=\langle u_t u_{t+\tau}\rangle/\{\langle u^2{}_t\rangle\langle u^2{}_{t+\tau}\rangle\}^{1/2},$$

it follows that

$$R(t)=\{R'+abR''\}/\{(1+a^2)(1+b^2)\}^{1/2},$$

where

$$R'=\langle u'_t u'_{t+\tau}\rangle/\{\langle u_t'^2\rangle\langle u'_{t+\tau}{}^2\rangle\}^{1/2},\quad R''=\langle u_t''u''_{t+\tau}\rangle/\{\langle u''_t{}^2\rangle\langle u''_{t+\tau}{}^2\rangle\}^{1/2}$$

$$a^2=\langle u_t''^2\rangle/\langle u'_t{}^2\rangle,\qquad b^2=\langle u''_{t+\tau}{}^2\rangle/\langle u'_{t+\tau}{}^2\rangle,$$

while

$$\langle u'_t u''_t\rangle=\langle u'_{t+\tau}u''_{t+\tau}\rangle=\langle u'_t u''_{t+\tau}\rangle=\langle u_t''u'_{t+\tau}\rangle=0\quad [6].$$

If u' is the turbulent fluctuations and u'' the wave currents, then $R''=\cos 2\pi t/T$. For example, put $a=b=1$ then R is as follows:

Table 4. Apparent auto-correlation coefficient in the presence of waves.

R'' \ R'	0.5	0	−0.5
1	0.75	0.5	0.25
0	0.25	0	−0.25
−1	−0.25	−0.5	−0.75

Thus in the presence of waves, the auto-correlations may be modified so that they become considerably larger or smaller than the true correlations due to the turbulence alone by the phase angle of waves. The high and low values of $R(t)$ shown in Figs. 6 and 7 are probably to be accounted for in this way.

Now let the vertical stability of water E, then the equation of motion of the water particle is given by $d^2z/dt^2=-Egz/\rho$ [7], then the period of oscillation is given by $T=2\pi(\rho/Eg)^{1/2}$. The vertical distributions of temperature are shown in Table 5.

Table 5. Water Temperature °C.

Date	Time	Depth (m) 0	1	3	5	7
Mar. 21	10 h	10.3	10.47	10.45	10.34	10.41
	10 h 36 m	10.1	10.62	10.55	10.46	10.44
Mar. 25	7 h 49 m	10.3	12.77	12.75	12.75	12.77
	8 h 17 m	13.6	13.13	13.14	13.14	13.12
	8 h 51 m	13.2	13.16	13.27	13.23	13.22

Unfortunately the chlorinity was not observed, so the data of a similar station and season published in 1937 by the Imperial Fisheries Experimental Station were used. Putting $t=13.1$°C, $S=33.33‰$ at the surface and $t=13.6$°C, $S=33.86‰$ at

10 m layer, then $E=0.3\times10^{-6}$ c.g.s., $\rho=1.025$ gr/cm^3 and $T=6.1$ min. Putting $t=10.47$°C at 1 m layer and $t=10.45$°C at 3 m depth as observed on March 21 and $S=33.33$‰, the stability is 1.8×10^{-6} and $T=2.5$min. Thus it may be sometimes $E=10^{-6}$ but it is usually estimated of the order of magnitude of the stability as 10^{-7}. Further, TAYLOR showed that stable internal waves are possible if $R_i>1/4$, where $R_i=\frac{g}{\rho}E\Big/\left(\frac{dU}{dz}\right)^2$ [6]. Assuming $dU/dz=1/50$ or $1/100$, we have the values of R_i.

Table 6. Value of TAYLOR criterion.

dU/dz	E	
	10^{-6}	10^{-7}
1/50	0.24	0.24×10^{-1}
1/100	9.6	0.96

So at the layer where dU/dz is small, mid-depths, the TAYLER criterion would suggest that stable internal waves are possible of which period T is estimated of the order of a few minutes as $E=10^{-6}$. Such a wave was not observed at 3 m depth but the observed period was 20 sec as the averaging time of observation was only 4 minutes. It was at the bottom layer that the other periodic fluctuations such as 2 or 3 min were observed and there dU/dz might not be small so the internal waves could not exist. But if $E=10^{-6}$, there might be such internal waves. We consider these long-period current fluctuations to have been caused by the turbulence if they were not caused by internal waves.

By an analogy of Strouhal's experiments [8], there may be periodic fluctuations of currents connected with the instability of vortex sheets, and its period is expressible by $T=5\,d/U$, where d is the diameter of an obstructor. Putting $U=19.9$ cm/sec $T=20$ sec and $U=10.4$ cm/sec $T=2.5$ min, we have $d=79.6$ cm and 312 cm respectively. It is estimated of the order of magnitude of the draft of the ship as 2 m or less and the roughness of the uneven bottom as the above order, a few meters, for the bottom topography is very complicated. Or the longer oscillation may be caused by secondary undulation of ocean tides, because in the vicinity of the station, there are many small bays whose period of oscillation is a few minutes.

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