SHORTER CONTRIBUTION

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On the Mean Spectral Distribution of the Height of 500-mb Isobaric Surface along the 50°N Latitude Circle

by

M. Iida, N. Nakamura and S. Kubota

Meteorological Research Institute

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1. Introduction

For the purpose of offering the materials for studying the relation between the various phenomena of different scales, the significance of mean values and the structure of large-scale turbulence in the atmosphere, we calculated the spectral distribution of disturbances in the atmosphere for one-day, and five-, ten-, twenty- and thirty-day mean values of height of isobaric surface, and investigated the seasonal variation of each region in the spectral distribution.

We calculated the thirty-six term Fourier coefficients of isobaric heights along 50°N latitude circles, using the 500-mb charts in the Historical Weather Maps of 1949.

2. Results

The curves of the spectral distribution of every day are complicated, and sometimes predominance of small scales is seen. So, to see how many spectral distributions of five-day means are necessiated in order to smooth such predominance of small-scale disturbance, we made mean distribution by adopting five, ten, ……, and forty numbers of distributions as shown in Fig. 1 (logarithmic scale and figures along each curve indicate the number of data). It can be seen from this figure that even when a remarkable predominence appears somewhere in the data of five-day mean spectral distibution, it is not always followed by the spectrum which cancels it.

Next we investgated the seasonal mean spectral distributions of isobaric height. In our case the whole year is divided into four seasons, i.e., winter, spring, summer and autumn, the period of each being determind by the zonal indices indicated by the difference between the mean heights of isobaric surface along 50°N and 30°N latitude circles. The results obtained thus are shown in Fig. 2 (a, b, c, d) respectively, in which small circles indicate the mean spectral distribution for one-day values and small triangles correspond to those for five-day mean values. Looking over the inclination of each curve in those figures, it will be noticed that the spectral domain of the atmosphere can be divided into two parts in substance, i.e., the part of small inclination or horizontal and the part of large inclination of curves of spectral distribution ($Z \propto n^{-7/3}$; Z is the height of isobaric

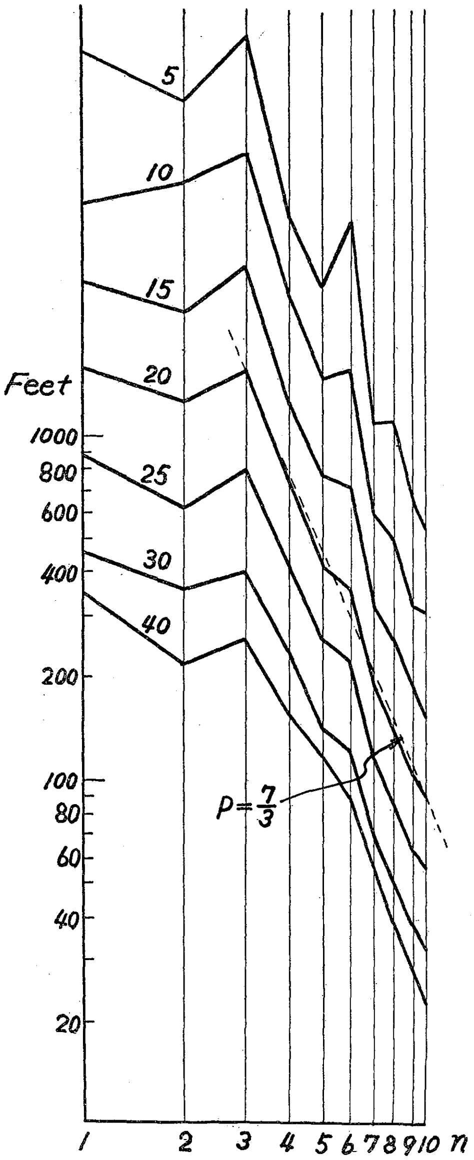

Fig. 1. Mean spectral distribution obtained from the 5, 10, ..., 40 individual distribution of five-day mean values.

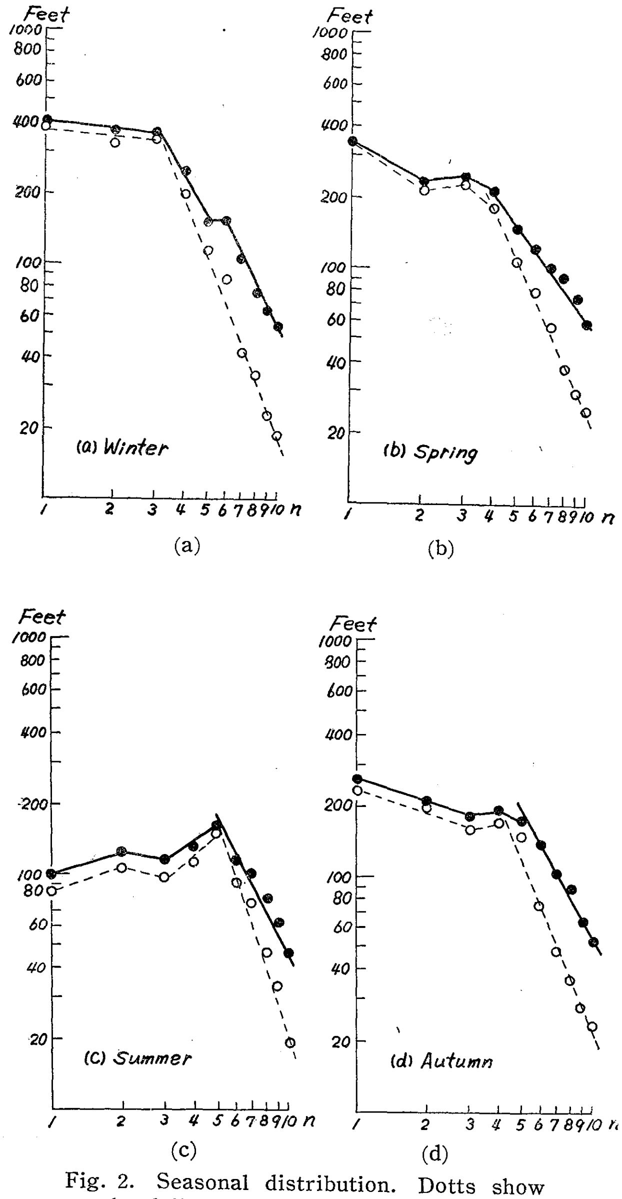

Fig. 2. Seasonal distribution. Dotts show the daily values, and small circles the five-day values.

suaface and n is the wave number of its component-waves). Hereafter we call the first part the first region and the second the second region.

In the following let us explain some features of the two regions. It is in the nature of things that the rate of decrease of each amplitude by five-day mean operation increases with the wave-number as seen in Fig. 2. However, even if the time interval of mean may be increased more than five-day, no remarkable change can be seen in the first region, and in the second the absolute values of amplitudes of each component wave may decrease but the inclination of curves is not changed, as shown in Fig. 3. These facts may suggest that the atmosphere expressed by five-day mean values or more is difficult to treat as the combination of propagating dispersive waves like Rossby waves. The fact stated here is related to the idea that the time mean may be suitable for the atmosphere considered from a thermo-dynamical standpoint in seasonal weather forecasting.

Then, to see the relation between the scale of disturbances and the frequency of its variation, we have put

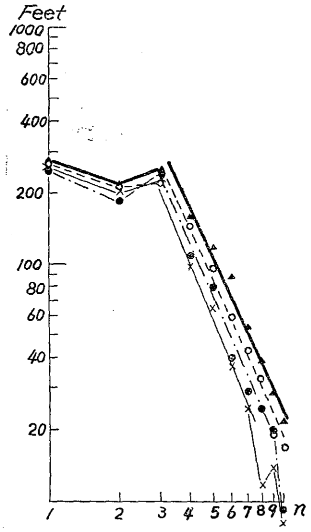

Fig. 3. Mean spectral distribution of various mean values.
○ five-day (40) ▲ ten-day (20).
● twenty-day (10) × thirty-day (6) mean values.
[figures in paren thesis indicate the nnmber of data used]

$$z=z_0+z_1+\cdots\cdots+z_n+\cdots\cdots=\sum_{n=0}^{\infty}z_n$$

$$=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\Big(a_{n,\nu}(\varphi)\cos\nu t\cos n\lambda+b_{n,\nu}(\varphi)\cos\tau t\sin n\lambda$$

$$+c_{n,\nu}(\varphi)\sin\nu t\cos n\lambda+\alpha_{n,\nu}(\varphi)\sin\nu t\sin n\lambda\Big),$$

$$z_n^2=\Big(\sum_{\nu=0}^{\infty}a_{n,\nu}(\varphi)\cos\nu t+\sum_{\nu=1}^{\infty}c_{n,\nu}(\varphi)\sin\nu t\Big)^2$$

$$+\Big(\sum_{\nu=0}^{\infty}b_{n,\nu}(\varphi)\cos\nu t+\sum_{\nu=1}^{\infty}\alpha_{n,\nu}(\varphi)\sin\nu t\Big)^2,$$

$$\overline{z}_n^2=\Big(\sum_{\nu=0}^{\infty}a_{n,\nu}(\varphi)\overline{\cos\nu t}+\sum_{\nu=1}^{\infty}c_{n,\nu}(\varphi)\overline{\sin\nu t}\Big)^2+\Big(\sum_{\nu=0}^{\infty}b_{n,\nu}(\varphi)\overline{\cos\nu t}+\sum_{\nu=1}^{\infty}\alpha_{n,\nu}(\varphi)\overline{\sin\nu t}\Big)^2,$$

where

$$\overline{\cos\nu t}=\cos\nu\Big(t-\frac{\alpha-1}{2}\Big)+\cdots\cdots+\cos\nu t+\cos\nu\Big(t+\frac{\alpha-1}{2}\Big),$$

$$\overline{\sin\nu t}=\sin\nu\Big(t-\frac{\alpha-1}{2}\Big)+\cdots\cdots+\sin\nu t+\sin\nu\Big(t+\frac{\alpha-1}{2}\Big),$$

and the mean distribution is assumed to be obtained from the following expressions:

$$\langle z_n^2\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}z_n^2\,dt=\sum_{\nu=0}^{\infty}\Big(a_{n,\nu}(\varphi)^2+b_{n,\nu}(\varphi)^2+c_{n,\nu}(\varphi)^2+d_{n,\nu}(\varphi)^2\Big),$$

$$\langle\overline{z}_n^2\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\overline{z}_n^2\,dt=\sum_{\nu=0}^{\infty}F(\nu,\alpha)^2\Big(a_{n,\nu}(\varphi)^2+\cdots\cdots+d_{n,\nu}(\varphi)^2\Big),$$

where

$$F(\nu,\alpha)=\sin\frac{\alpha\nu}{2}\Big/\sin\frac{\nu}{2},$$

where z is the height of isobaric surface,

φ: latitude,
λ: longitude
ν: frequency
α: the period of averaging.

Accordingly it will be concluded the spectral distribution against ν of every component-wave is similar for each, which is shown in Fig. 4. Namely, if the slope of these curve differs, the slope of spectral distribution against n accordingly should differ. From this consideration it can be said that the frequency of variation is not proportional statistically to the scales, at least in the region $10 \geqq n \geqq 4$. So we should deal with each region of the atmosphere individually, like the first and the second, also as to the frequency of variation.

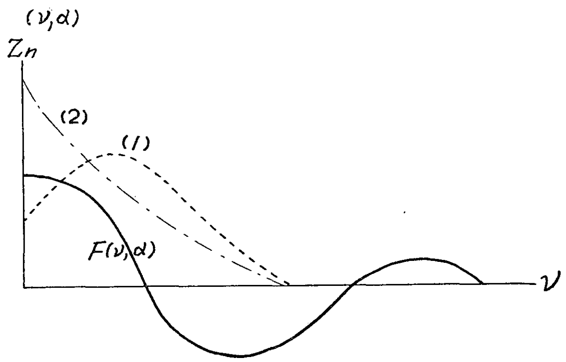

Fig. 4. Models of spectral distribution against ν and curve of $F(\nu, \alpha)$, e.g., curve (2) for large scale and curve (1) for small scale.

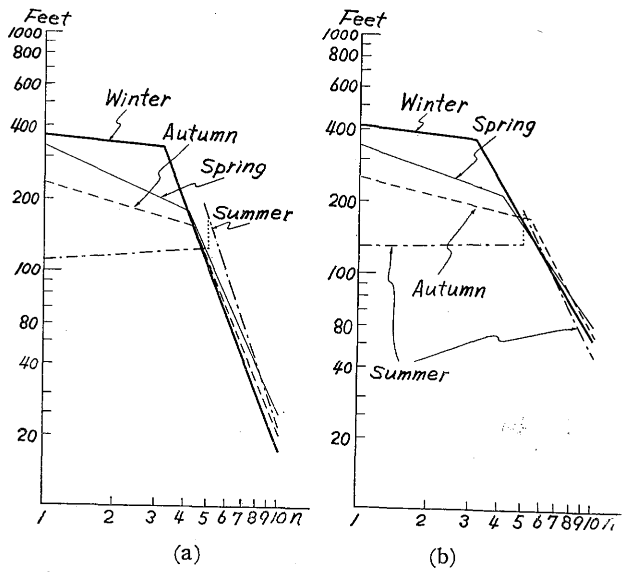

Fig. 5. Models of seasonal distribution. Left for daily distribution and right for the five-day mean values.

Next we investigated the seasonal variation of spectral distribution, using the models which are obtained from Fig. 2, (a), (b), (c), (d) and they are in Fig. 5. The illustration on the left in Fig. 5 is the mean spectral distributions of every-day data and the right one corresponds to the mean distributions of the five-day mean values. However, no remarkable difference can be noticed between the two distributions. Now, looking at the figures, it is found that seasonal variation is distinct in the first region but indistinct in the second, except for a small variation seen in the inclination of the distribution curves. Seasonal variation in the first region is as follows: the curve in winter is horizontal and the amplitude of each component wave is largest, but the curve becomes a little inclined and the region larger and larger in spring, till in summer the curve becomes horizontal again, having the largest region, and further with the progress of season the reverse process goes on through autumn until winter. In order to express the above explanation a little more quantitatively, we calculated the meridionnal components of the kinetic energy $1/2\cdot\rho v^2$ for the first region, where

$$v=\frac{g}{fa\cos\varphi}\frac{\partial z}{\partial\lambda},$$

and

$$z=A_0(\varphi,t)+\sum_{n=1}^{i}\left\{A_n(\varphi,t)\cos n\lambda+B_n(\varphi,t)\sin n\lambda\right\},$$

$$i=\begin{cases}3\cdots\cdots\text{winter}\\4\cdots\cdots\text{spring}\\6\cdots\cdots\text{summer}\\5\cdots\cdots\text{autumn.}\end{cases}$$

Namely,

$$\frac{1}{2\pi}\int_0^{2\pi}\rho v^2\,d\lambda=\frac{1}{2}\rho\frac{g^2}{f^2a^2\cos^2\varphi}\sum_{n=1}^{i}n^2(A_n^2+B_n^2),$$

gives the following table.

Season	i	$\sum_{n=1}^{i} n^2(A_n^2+B_n^2)$
Winter	3	1.7×10^4 feet^2
Spring	4	$1.9\times$ 〃 〃
Summer	6	$2.0\times$ 〃 〃
Autumn	5	$1.9\times$ 〃 〃

This table tells that the meridional component of the kinetic energy is relatively conservative regardless of the seasonal variation of the first region.

Finally, only the general aspects of the seasonal variation of spectral distribution of 500-mb isobaric surface along 50°N latitude circle is touched upon in the present paper. Further investigation of their details will be reported in other papers.

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