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A Computation of the Effect of Coalescence on Raindrop Size-Distributions

by

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1. Introduction

Since the observation of raindrop size was begun, to evaluate the effect of coalescence on the size-distribution of raindrops has been one of the most interesting themes in cloud physics, and in the beginning of it attention was centered on the problem of the so-called group distribution of raindrops. Recently a few evaluations of the effect of coalecence of raindrops have been made in relation to the development of radar rainfall observations. One of them has been made in 1954 by RIGBY, GUNN and HITSCHFELD[1], and the other in 1954 by MASON and RAMANADAHAM[2]. After calculations, the former concluded that drop size distributions may not vary so much with height and that in weaker-than-light rain the effect will be negligible, but the latter emphasized that the distributions of raindrops may be modified by a considerable magnitude even only by coalescence process. The equations which they have integrated numerically are the same and have been simplified into a form of successive calculation so as to facilitate integration.

The computation of the present author has been based on a fundamental coalescence equation different from that above, which has a form of integro-differential equation as introduced by SCHUMANN[3].

2. Fundamental coalescence equation and method of computation

The notations used in this paper are as follows:

$f(x, t)$ =volume distribution function, a quantity such that the concentration of the drops having masses x (or X_i) to $x+dx$ (or X_{i+1}) is represented by $f(x, t)dx$ (or F_i), t=time.

V_x (or V_i) =falling velocity of a raindrop having mass x (or X_i),

$c(x, y)$ (or $C_{i,j}$) =collision factor between two drops having masses x (or X_i), and y (or X_j),

h (or H) =altitude.

From the mass-conservation law, the net spatial increment of volume distribution function f during a unit time interval will be given by a summation of the time differentiation $\partial f/\partial t$, which is caused by mutual collisions in the system, and of the convergence of particle flow into the system. Assuming that the physical and meteorological conditions at any level be stationary with time and horizontally homogeneous, the continuity equation of drop concentration becomes

$$\frac{\partial f}{\partial t}-\left(\frac{\partial f}{\partial h}\right)v=0. \tag{1}$$

Furthermore, the equation (1) involves an assumption that the falling velocities of raindrops, having a constant mass x, should not vary. $\partial f/\partial t$, the partial time differentiation of f, which means a rate of variation of distribution function f in the closed system, will be expressed by the difference between the number of new drops composed of smaller two drops by collisions, which is given by

$$(2) \qquad \frac{1}{2}\int_0^x f(y,\ t)f(x-y,\ t)c(y,\ x-y)dy,$$

and the number of lost drops by capture given by

$$(3) \qquad f(x,\ t)\int_0^\infty f(y,\ t)c(x,\ y)dy,$$

where $c(x, y)$ is the coalescence function, defined so that the average number of coalescences per unit volume between raindrops of mass x to $x+dx$ and raindrops of mass y to $y+dy$ in a unit time interval is equal to $f(x, t)f(y, t)c(x, y)dxdy$, and depends on collision efficiency, coalescence efficiency (fraction of collisions resulting in aggregation), the sweep cross-section and the speed difference between them.

The function $c(x, y)$ is chosen to be

$$c(x,\ y)=E(x,\ y)|V_x-V_y|s(x,\ y),$$

where $E(x, y)$ is LANGMUIR's coefficient ($\fallingdotseq 1$), and $s(x, y)$ is the sweep cross-section.

In this paper, no consideration has been taken into account as to the effect on the size-distribution of raindrops of such factors as evaporation of drops, accretion of cloud droplets on raindrops, break-up or bouncing at the collison of drops.

Thus, from equations (1), (2) and (3) the coalescence equation becomes

$$(4) \qquad \frac{\partial f}{\partial h}\cdot V_x-\frac{1}{2}\int_0^x f(y,\ t)f(x-y,\ t)c(y,\ x-y)dy$$
$$+f(x,\ t)\int_0^\infty f(y,\ t)c(x,\ y)dy=0.$$

In order to facilitate the numerical integration the equation (4) will be now rewritten into a form of the finite differences of 20 grades and 19 steps.

Let

$$(5) \qquad \triangle F_i=\frac{\triangle H}{V_i}\left\{\frac{1}{2}\sum_{j=1}^{i=i-1} F_jF_{i-j}C_{i-j,\ j}-\sum_{j=1}^{20}F_iF_jC_{j,i}\right\},$$

where i, j are numbers of the steps of the finite differences, each step being defined so as to have an equal volume interval of drops between adjacent grades, i.e. $X_{i+1}-X_i=X_i-X_{i-1}$, etc. And also the volume of the smallest drops are limited to that of the interval correspond to a step. The initial distribution function F_i thus given as the step function comes to have the conservative steps, since no new grade different from the initial grades of drop volume arise in collisions.

The minimum drop volume was defined to be 0.25 mm^3. It is obvious that, through these formulations of initial distribution, the argument would not lose its generality and reality. Of course, actual distributions of raindrops in the atmosphere have not always formal and smoothed distribution curves, such as exponential curves formulated by MARSHALL and PALMER in 1948[5]. By means of these particular formulations of initial distribution curve in the step function, the total num-

Table 1.

20	6.46													0.000 0.000
⋮	⋮											…	…	…
j	F_j										…	$C_{i,j}$ $F_iF_jC_{i,j}$	…	…
…	…									…	…	…	…	33.18 17251
8	112.4								0.000	3.236 29214	…	…	…	36.41 26480
7	166.2							0.000	3.403 63566	6.641 88681	…	…	…	39.82 43504
6	251.6						0.000	3.479 ×10 14548	6.883 19460	10.05 20312	…	…	…	43.33 70526
5	404.5					0.000	3.812 ×10 38789	7.291 49021	10.71 48657	13.92 45223	…	…	…	47.30 12374
4	717.6				0.000	3.775 $\times 10^2$ 10956	7.590 10702	11.06 13195	14.45 11656	17.74 10228	…	…	…	51.15 ×10 23740
3	1478			0.000	4.080 $\times 10^2$ 43272	7.866 47025	11.69 43468	15.17 37270	18.59 30878	21.71 25775	…	…	…	55.41 ×10 52945
2	3704		0.000	4.458 $\times 10^3$ 24411	8.552 22734	12.36 18524	16.21 15110	19.70 12131	23.15 $\times 10^2$ 9640	26.25 7811	…	…	…	60.25 $\times 10^2$ 14426
1	3065 ×10	0.000	5.583 $\times 10^4$ 63398	10.10 45797	14.32 31502	18.25 22637	22.23 17145	25.81 13151	29.37 10120	32.81 $\times 10^3$ 8079	…	…	…	67.59 $\times 10^3$ 13371
j	F_j / F_i	3065 ×10	3704	1478	717.6	404.5	251.6	166.2	112.4	80.33	…	F_i	…	6.46
	i	1	2	3	4	5	6	7	8	9	…	i	…	20

$\varphi_i = \sum_{j=1}^{i-1} F_jF_{i-j} \times C_{j,i-j}$	$\times 10^5$ 24681	$\times 10^4$ 78037	51779	35343	25518	19461	15029	11630	$\times 10^3$ 93219	76029	…	φ_i	…	$\times 10^3$ 15924
$\chi_i = \frac{1}{2}\sum_{j=1}^{20} F_iF_jC_{i,j}$	0	$\times 10^4$ 63398	45797	33943	24910	19430	15132	11878	$\times 10^3$ 95529	78142	…	χ_i	…	$\times 10^3$ 16567
$\varphi_i - \chi_i$	$\times 10^5$ 24681	$\times 10^4$ 14639	5982	1400	608	31	−104	−248	10^3 −2310	−2113	…	$\varphi_i - \chi_i$	…	10^3 −643

ber of steps has become capable of reduction to a considerably small number, such as 20, without losing the reliablity of evaluation.

The falling velocities used in calculation are quoted from GUNN and KINZER's table[6]. The blank form ruled in a checker pattern having 20 segments on each side was used for the computation (see Table 1). Numbering the segments along the abscissa with $i=1, 2, 3, \cdots\cdots, 20$ and along the ordinate with $j=1, 2, 3, \cdots\cdots, 20$, the values of $F_iF_j \cdot C_{i,j}$ calculated in the second and the third terms of the equation (5) are inscribed into the segment (i, j) of the blank form correspond to Nos., i and j on the abscissa and ordinate respectively.

Since values of $C_{i,j}$ are determined uniquely from Nos. i and j, they may be inscribed previously in a corner of the segment (i,j) for convenience of calculation. These are quite symmetric with respect to i and j, so that the inscriptions need not be made in both sides of the diagonal line from (1, 1) to (20, 20) of the blank form. Then, the values of the second and the third term of equation (5) will be obtained by means of summation of the segments aslant and vertically respectively.

3. Results

A result of the computation is shown by B' on Fig. 1, in which curve A' is copied from RIGBY's paper and C' from MASON's. The curves A, B and C indicate the initial distribution of each calculation.

At a glance, it will be seen that the author's result shows the effect on the raindrop size-distribution to be considerably slight, and rather similar to that of RIGBY *et al* than to that of MASON *et al.* Finally, the author cannot but conclude from his results thus obtained that the variations of echo intensities developed in a falling pattern as usually observed on a radar scope are perhaps not caused only by coalescence process. In the actual atmosphere, processes such as accretion of cloud droplets on raindrops, evaporation of drops, horizontal mixing or advection, and up or down-drafts do often exist simaltaneously and come into the problem of distribution function.

These complicated effects might be of more real importance in the formation of actual storm raindrop distributions.

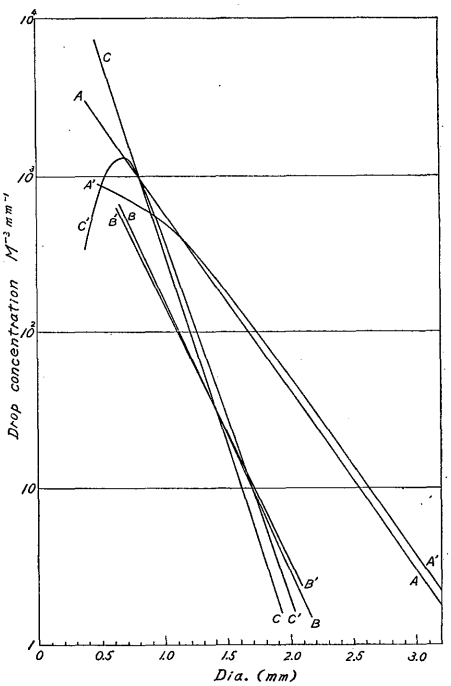

Fig. 1. The dushed curves show the distributions modified by only coalescence in 1 km fall.

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