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A Prediction for the Next Maximum of Sunspot Numbers

by

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Abstract

The magnitude and the date of the next maximum of Wolf's sunspot numbers are stochastically predicted. The most probable values of them are 118.4 and Jan. 1958 respectively and the uncertainty of them is indicated individually and jointly by a sort of probability distribution.

1. Introduction

In this year Wolf's sunspot numbers got their minimum. What amount the magnitude of the next maximum will attain? And when the maximum will come about? In order to get a definite prediction for the date of the maximum, it is better to predict directly the extremum rather than to do with the change of the monthly or yearly mean numbers. The extremums and their dates of sunspot numbers were defined for a series made by means of the moving average of tweleve months for the monthly mean values of Wolf's sunspot numbers. The extremums thus obtained and their dates represented by the decimal system taking a year for its unit are shown in Table 1.

Table 1. The extremums and their dates of Wölf's sunspot numbers.

t	Minimum Date	Minimum Magnitude x_{2t}	Maximum Date	Maximum Magnitude y_{1t}	Period Min.-Min. x_{1t}	Period Min.-Max. y_{2t}
	1755.2	—				
1	1766.4	10.9	1769.8	116.4	11.2	3.4
2	1775.5	7.0	1778.4	158.7	9.1	2.9
3	1784.8	9.5	1788.1	141.2	9.3	3.3
4	1798.3	2.9	1805.1	49.2	13.5	6.8
5	1810.6	0.0	1816.4	48.7	12.3	5.8
6	1823,3	0.1	1829.9	71.6	12.7	6.6
7	1833.9	7.1	1837.3	141.9	10.6	3.4
8	1843.6	10.4	1848.1	131.9	9.7	4.5
9	1856.0	3.2	1860.1	98.1	12.4	4.1
10	1867.2	5.1	1870.7	140.9	11.2	3.5
11	1878.8	2.1	1883.9	75.6	11.6	5.1
12	1890.2	4.5	1894.1	89.1	11.4	3.9
13	1902.0	2.6	1906.5	63.5	11.8	4.5
14	1913.5	1.4	1917.6	105.7	11.5	4.1
15	1923.6	5.4	1928.3	78.2	10.1	4.7
16	1933.8	3.2	1937.3	120.7	10.2	3.5
17	1944.2	7.6	1948.8	150.3	10.4	4.6
0	1954.3	3.4	?	?	10.1	?

The last row in this table, the date of minimum 1954.3, the minimum value $x_{20}=3.4$ and the min.-min. period $x_{10}=10.1$ which will give the initial conditions in our prediction, were determined on the basis of the observations by the Tokyo Astronomical Observatory at Mitaka.

2. Prediction formula

Let us use the notations x_{1t}, x_{2t}, y_{1t} and y_{2t} in the meaning indicated in Table 1. The available data shown in Table 1 are rather a small sample and we should use an exact sampling prediction formula derived by one of the authors [1] for a normal stationary stochastic process. The statistical stationarity of our data may be assumed and the normality of probability distributions may be also assumed for x_{1t}, x_{2t} and y_{1t}, but that for y_{2t} is somewhat doubtful; thus we use $\eta_{2t}=\log_{10} y_{2t}$ whose frequency distribution is more close to a normal one than that of y_{2t} itself.

After examining the various factors which seem to have some correlation to the predictands y_{1t} and η_{2t} we found that the following correlations are important:

$$\hat{\rho}(y_1, x_1)=-0.796^{**}, \qquad \hat{\rho}(y_1, x_2)=\ \ 0.755^{**},$$
$$\hat{\rho}(\eta_2, x_1)=\ \ 0.730^{**}, \qquad \hat{\rho}(\eta_2, x_2)=-0.578^{*},$$
$$\hat{\rho}(y_1, \eta_2)=-0.779^{**}, \qquad \hat{\rho}(x_1, x_2)=-0.702^{**},$$

where, for instance, $\hat{\rho}(y_1, x_1)$ is the estimated correlation coefficient between y_{1t} and $x_{1t}(t=1, 2, \cdots\cdots, N;\ N=17)$ and the notations * and ** indicate that the coefficient is statistically significant on the 5% and 1% level of significance respectively. Other correlations are small and especially the autocorrelation of either y_{1t} or η_{2t} is negligible. The multiple correlation coefficient of y_{1t} and η_{2t} with x_{1t} and x_{2t} resulted in

$$\hat{\rho}(y_1 \cdot x_1, x_2)=0.824^{**} \text{ and } \hat{\rho}(\eta_2 \cdot x_1, x_2)=0.736^{**},$$

respectively, and even if we add other factors the significance of these coefficients does not increase owing to the reduction of degrees of freedom.

Thus we can use the following prediction formulas (OGAWARA [1], [2], [3])

i) Prediction formula for y_{10}

$$(1) \qquad \bar{y}_1+\hat{a}_1(x_{10}-\bar{x}_1)+\hat{a}_2(x_{20}-\bar{x}_2)$$
$$\pm [F^1_{N-3}(2\alpha)]^{1/2}\cdot\Big[\frac{1}{N-3}\sum_{t=1}^{N}\{y_{1t}-\bar{y}_1-\hat{a}_1(x_{1t}-\bar{x}_1)-\hat{a}_2(x_{2t}-\bar{x}_2)\}^2$$
$$\cdot\Big\{1+\frac{1}{N}+\sum_{p,q=1}^{2}c^{pq}(x_{p0}-\bar{x}_p)(x_{q0}-\bar{x}_q)\Big\}\Big]^{1/2},$$

where

$$\bar{x}_p=\sum_{t=1}^{N}x_{pt}/N \quad (p=1, 2), \quad \bar{y}_1=\sum_{t=1}^{N}y_{1t}/N,$$
$$c_{pq}=\sum_{t=1}^{N}(x_{pt}-\bar{x}_p)(x_{qt}-\bar{x}_q), \quad c'_{1p}=\sum_{t=1}^{N}(y_{1t}-\bar{y}_1)(x_{pt}-\bar{x}_p) \quad (p, q=1, 2),$$
$$c_{11}\hat{a}_1+c_{12}\hat{a}_2=c'_{11}, \quad c_{21}\hat{a}_1+c_{22}\hat{a}_2=c'_{12},$$
$$(c^{pq})=(c_{pq})^{-1},$$

and $F^1_{N-3}(2\alpha)$ is the $100\times 2\alpha\%$ point of the F-distribution with 1 and $N-3$ degrees of freedom. Let us denote the two values of (1) by

$$m(y_1, x_1, x_2) \pm [F(2\alpha)]^{1/2}\sigma(y_1, x_1, x_2),$$

where $y_1=(y_{1t}\,;\ t=1, 2, \cdots\cdots)$ is a stochastic variable and $x_1=(x_{1t}\,;\ t=0, 1, 2, \cdots\cdots)$ and $x_2=(x_{2t}\,;\ t=0, 1, 2, \cdots\cdots)$ are fixed variates, and denote the relation between y_{10} and α given by the equations

$$(2)\qquad \begin{aligned} y_{10} &= m(y_1, x_1, x_2) - [F(2\alpha)]^{1/2}\sigma(y_1, x_1, x_2) && \text{for } 0<\alpha\leqq 1/2 \\ &= m(y_1, x_1, x_2) + [F(2(1-\alpha))]^{1/2}\sigma(y_1, x_1, x_2) && \text{for } 1/2\leqq\alpha<1 \end{aligned}$$

by

$$(3)\qquad \alpha=H(y_{10}\,;\ y_1,\ x_1,\ x_2).$$

The equation (3) is a cumulative distribution function of y_{10} for fixed y_1, x_1 and x_2 and define a stochastic prediction for y_{10}. The stochastic prediction for y_{10} may be expressed also by the density function

$$(4)\qquad \frac{d}{dy_{10}}H(y_{10}\,;\ y_1, x_1, x_2).$$

ii) Prediction formula for y_{20}

Quite similarly to the case of y_{10} the stochastic prediction for η_{20} is given by

$$\alpha=K(\eta_{20}\,;\ \eta_2,\ x_1,\ x_2).$$

Transforming it by $\eta_{20}=\log y_{20}$, we get the stochastic prediction for y_{20}. The density function may be easily obtained from the cumulative distribution by a graphical method.

iii) Joint prediction formula for y_{10} and y_{20}

Firstly, the joint prediction formula for y_{10} and η_{20} is given by

$$(5)\qquad \sum_{i,j=1}^{2} a^{ij}\Big\{y_{i0}'-\bar{y}_i'-\sum_{p=1}^{2}\hat{a}_{ip}(x_{p0}-\bar{x}_p)\Big\}\Big\{y_{j0}'-\bar{y}_j'-\sum_{q=1}^{2}\hat{a}_{jq}(x_{q0}-\bar{x}_q)\Big\}$$
$$=F^2{}_{N-4}(\alpha)\cdot\frac{2}{N-4}\Big\{1+\frac{1}{N}+\sum_{p,q=1}^{2}(x_{p0}-\bar{x}_p)(x_{q0}-\bar{x}_q)\Big\},$$

where

$$y_{1t}'=y_{1t},\quad y_{2t}'=\eta_{2t}\quad (t=0, 1, 2, \cdots\cdots),$$
$$\bar{y}_1'=\sum_{t=1}^{N}y_{1t}/N,\quad \bar{y}_2'=\sum_{t=1}^{N}\eta_{2t}/N,$$
$$c_{pq}=\sum_{t=1}^{N}(x_{pt}-\bar{x}_p)(x_{qt}-\bar{x}_q)\quad (p, q=1, 2),$$
$$c'_{ip}=\sum_{t=1}^{N}(y_{it}'-\bar{y}_i')(x_{pt}-\bar{x}_p)\quad (i=1, 2\,;\ p=1, 2),$$
$$\sum_{p=1}^{2}c_{pq}\hat{a}_{ip}=c_{iq}'\quad (i=1, 2\,;\ q=1, 2),$$
$$(c^{pq})=(c_{pq})^{-1},$$
$$(a^{ij})=(a_{ij})^{-1},$$
$$a_{ij}=\sum_{t=1}^{N}\Big\{y_{it}'-\bar{y}_i'-\sum_{p=1}^{2}\hat{a}_{ip}(x_{pt}-\bar{x}_p)\Big\}\Big\{y_{jt}'-\bar{y}_j'-\sum_{q=1}^{2}\hat{a}_{jq}(x_{qt}-\bar{x}_q)\Big\}$$
$$=\sum_{t=1}^{N}(y_{it}'-\bar{y}_i')(y_{jt}'-\bar{y}_j')-\sum_{p=1}^{2}\hat{a}_{ip}c_{jp}'$$
$$=\sum_{t=1}^{N}(y_{it}'-\bar{y}_i')(y_{jt}'-\bar{y}_j')-\sum_{q=1}^{2}\hat{a}_{jq}c_{iq}'.$$

The probability that the point (y_{10}', y_{20}') lies in the ellipse (5) is $1-\alpha$. Thus, the joint stochastic prediction for y_{10} and η_{20} may be represented by a family of ellipses (5) corresponding to various values of α and that for y_{10} and y_{20} is obtained by the transformation $y_{10}'=y_{10}$, $y_{20}'=\log y_{20}$ from the equation (5).

3. Prediction

From our data indicated in Table 1 we get the following prediction equations.

i) The prediction of the magnitude y_{10} of the next maximum:

(6) $$y_{10}=118.37\pm[F_{14}^{1}(2\alpha)]^{1/2}\times 24.60.$$

ii) The prediction of the date:

(7) $$\eta_{20}=\log y_{20}=\log 3.78\pm[F^{1}_{14}(2\alpha)]^{1/2}\times 0.08659.$$

The required date is given by $1954.3+10^{\eta_{20}}$.

iii) The joint prediction of y_{10} and y_{20}:

(8) $$0.0001858(y_{10}-118.37)^2+0.04918(y_{10}-118.37)(\log y_{20}-0.5777)$$
$$+15.000(\log y_{20}-0.5777)^2=F_{13}^{2}(\alpha)\times 0.18974.$$

From these equations, the "mean value" of the magnitude of the next maximum is 118.4 and that of y_{20} is 3.8, consequently the expected date of the maximum is 1954.3+3.8=1958.1.

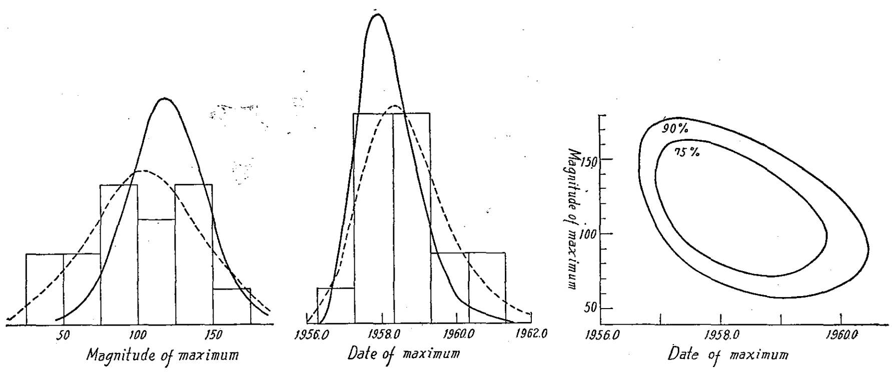

Fig. 1. Stochastic prediction of the next maximum of sunspot numbers. Full lines are the "probability densities" of the predictands. Dotted lines are the "absolute probability densities". The percentages assigned to the closed curves are the probabilities that the point (y_{10}, y_{20}) falls in the respective curves. The histograms show the respective empirical frequency distributions.

The stochastic predictions are shown by full lines in Fig. 1, where the individual predictions are expressed by the "density functions" and the joint prediction by the transformed ellipses. The dotted line corresponds to the estimation of the absolute distribution of each quantity and was calculated, for instance as to y_{10}, by the formula

(9) $$\bar{y}_1\pm[F_{N-t}^{1}(2\alpha)]^{1/2}\cdot\left[\frac{N+1}{N-1}s_1^2\right]^{1/2},$$

where $\bar{y}_1=\sum_{t=1}^{N}y_{1t}/N$, $s_1^2=\sum_{t=1}^{N}(y_{1t}-\bar{y}_1^2)/N$, in stead of (1), and the formulae similar to (2)~(4).

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References

[1] OGAWARA, M. 1955; A General Stochastic Prediction Formula, Pap. Met. Geophys., 5, p. 193.

[2] OGAWARA, M. and Collaborators, 1954: Prediction by a Small Sample, Journ. Met. Res., 6, p. 172. (in Japanese)

[3] OGAWARA, M. and Collaborators, 1954: Stochastic Limits for the Maximum Possible Amount of Precipitation, Pap. Met. Geophys., 5, p. 8.