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Kinematics of Meandering and Blocking Action of the Westerlies

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Abstract

Using the velocity profile with maximum zonal wind at about 26.5° latitude, and assuming the conservation of the absolute vorticity, the author derived the stream function for the finite perturbation motion. The method of analysis used is quite similar with that given in papers by HAURWITZ, CRAIG, NEAMTAN and HφILAND. The streamlines associated with small stationary perturbations and the streamlines associated with relatively large stationary perturbations are drawn as examples. In the former case, the streamlines surround the pole and the effects of the disturbance appear as north-southerly undulations or meanderings of the streamlines. In the latter case, the basic westerly flow splits into two separate branches. Each of these branches transports an appreciable mass, and a dynamic high occurs just south of the northern current branch and a dynamic low just north of the southern branch. This may offer a simple explanation for the kinematics of blocking action on the rotating spherical earth.

In a paper C.-G. ROSSBY [1] has discussed the effect of the latitudinal variation of the CORIOLIS force on the propagation of atmospheric disturbance. The direction of the wave propagation is assumed to coincide with the W-E direction which serves as x-axis. He assumes further that the disturbance is independent of the y-coordinate, that is of the direction normal to the direction of propagation. B. HAURWITZ [2a, 2b] extended the discussion where it is not assumed that the disturbance is independent of the y-coordinate, and finally he also discussed the wave motions taking the spherical shape of the earth into account. In his famous paper, HAURWITZ assumed a special profile for zonal circulation as:

$$U=\alpha a\cos\phi,$$

where ϕ is the latitude increasing towards the north, a the earth's radius, U the fundamental velocity of the zonal circulation of the atmosphere, and α is assumed as constant for the sake of simplicity. This velocity profile is of course unfavourable for the present purpose, for it shows the air motion relative to the earth with constant angular velocity and it does not show the maximum westerlies in the middle latitudes. His investigation has been extended in this particular. Here the velocity of undisturbed zonal current is assumed as

$$U=\frac{a\Omega}{16}\cos^3\phi(1+3\sin^2\phi), \tag{1}$$

where Ω is the angular velocity of the earth's rotation. The stream function Ψ for this basic flow is given by

$$\Psi=a^2\Omega(15\sin\phi+10\sin^3\phi-9\sin^5\phi)/240. \tag{1'}$$

The velocity profile (1) can also be written as

$$U=\frac{a\Omega\cos\phi(1-\sin\phi)}{1+\sin\phi}\cdot\left(\frac{1+\sin\phi}{2}\right)^2\left(\frac{1+3\sin^2\phi}{4}\right),$$

so that near the pole ($\phi=90°$) this velocity profile is very similar with the well-known velocity profile given by ROSSBY [3] as:

$$U=\frac{a\Omega\cos\phi(1-\sin\phi)}{(1+\sin\phi)}. \tag{1a}$$

The maximum zonal wind is $0.07155a\Omega$ at about 26.5° latitude, and the wind speed at the equator is given by $0.0625a\Omega$. The velocity profile shown in Fig. 1 is of course not the best one, but it is more favourable than the profile used by HAURWITZ.

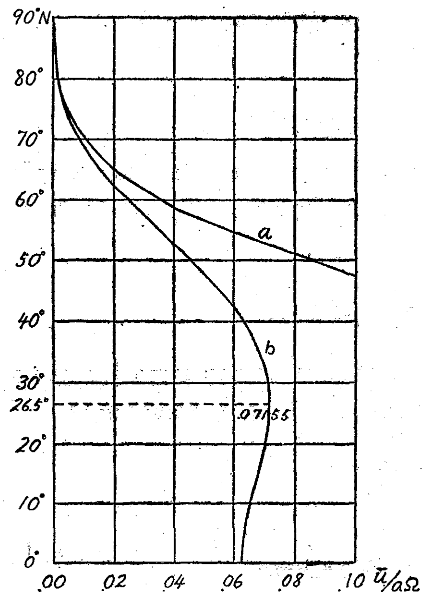

Fig. 1. Velocity profile of the undisturbed zonal motion.

The vertical component of absolute vorticity $\overline{Z}$ is then given by

$$\overline{Z}=-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\cdot U)+2\Omega\sin\phi,$$

or

$$\overline{Z}=\frac{\Omega}{16}(30\sin\phi+20\sin^3\phi-18\sin^5\phi). \tag{2}$$

Thus the absolute vorticity corresponding to the velocity profile is 2Ω at the north pole and zero at the equator.

On this zonal current a disturbance with velocity components towards east and north is superimposed. u' and v' may be considered so small that terms of higher order in the perturbation velocities or their derivatives can be neglected. The absolute vorticity is then given by

$$(2') \qquad Z=-\frac{\partial}{a\cos\phi\,\partial\phi}[(U+u')\cos\phi]+\frac{\partial v'}{a\cos\phi\,\partial\lambda}+2\Omega\sin\phi.$$

The atmosphere will be considered as incompressible for the sake of simplicity. Then equation of continuity may be written in the form

$$(3) \qquad \frac{1}{a\cos\phi}\left[\frac{\partial}{\partial\phi}(\cos\phi\cdot v')+\frac{\partial u'}{\partial\lambda}\right]=0,$$

where λ is the longitude, by a suitable choice of zero meridian $\lambda=0$. The form of equation (3) suggests the introduction of a stream function Ψ by the equations

$$(4) \qquad u'=\partial\psi/a\partial\phi, \qquad v'=-\partial\psi/a\cos\phi\,\partial\lambda.$$

The absolute vorticity (2′) then becomes

$$(5) \qquad Z=-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\cdot U)+2\Omega\sin\phi-\nabla^2\psi,$$

where

$$\nabla^2=\frac{\partial^2}{a^2\,\partial\phi^2}-\frac{\tan\phi}{a^2}\frac{\partial}{\partial\phi}+\frac{1}{a^2\cos^2\phi}\frac{\partial^2}{\partial\lambda^2}.$$

The absolute vorticity must be constant by small disturbances. If it is assumed that the air is disturbed while retaining its absolute vorticity and stationary disturbance is brought about. Then differentiating equation (5) with respect to time it follows that

$$(6) \qquad \frac{U}{a\cos\phi}\frac{\partial}{\partial\lambda}\left[\frac{\partial^2\psi}{a^2\partial\phi^2}-\frac{\tan\phi}{a^2}\frac{\partial\psi}{\partial\phi}+\frac{1}{a^2\cos^2\phi}\frac{\partial^2\psi}{\partial\lambda^2}\right]$$
$$+\frac{\partial\psi}{a^2\cos\phi\,\partial\lambda}\frac{\partial}{\partial\phi}\left[-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi.U)+2\Omega\sin\phi\right]=0.$$

It may be assumed that

$$(7) \qquad \psi(\lambda,\phi)=(A_m\sin m\lambda+B_m\cos m\lambda).F(\phi),$$

where m is the wave number, and A_m and B_m are arbitary constants, respectively. Substituting this expression (7) for ψ in the differential equation (6), it is found that $F(\phi)$ must satisfy the equation

$$\frac{d^2F}{d\phi^2}-\tan\phi\frac{dF}{d\phi}+\left(5\times 6-\frac{m^2}{\cos^2\phi}\right)F=0.$$

At the pole F must vanish, since the value of ψ must here be the same for every value of λ. This is only possible if F is an associated LEGENDRE polynomial,

$$F=\cos^m\phi\frac{d^mP_5(\sin\phi)}{d(\sin\phi)^m}=P_5^m(\sin\phi),$$

where

$$P_5(\sin\phi)=\frac{1}{8}(63\sin^5\phi-70\sin^3\phi+15\sin\phi).$$

The stream function for the perturbation motion is given by

(8) $$\psi=(A_m \sin m\lambda+B_m \cos m\lambda)P_5^m(\sin\phi).$$

The perturbation streamlines for $m=2$ has been shown in Fig. 62 in HAURWITZ's paper [2*b*]. The stream function for the total (disturbed+undisturbed) velocity

(9) $$\Psi+\psi=a^2\Omega(15\sin\phi+10\sin^3\phi-9\sin^5\phi)/240 \\ +(A_m \sin m\lambda+B_m \cos m\lambda)P_5^m(\sin\phi).$$

This final solution is derived under the assumption of small disturbance where terms of higher order in the perturbation velocities or their derivatives can be neglected. I shall now proceed to show that the final solution given by (8) strictly holds for finite or large disturbances.

Substituting the expressions (8) for ψ and (1) for U in (5), it is found that the vertical component of the absolute vorticity becomes

$$Z=\frac{\Omega}{8}(15\sin\phi+10\sin^3\phi-9\sin^5\phi)+\frac{30}{a^2}(A_m \sin m\lambda+B_m \cos m\lambda)P_5^m(\sin\phi),$$

or

(10) $$Z=\frac{30}{a^2}(\Psi+\phi).$$

Along the streamlines of the total velocity $\Psi+\psi=$const., the absolute vorticity is thus constant. Hence it is shown that the solution given by equation (9) is the exact stationary solution and it holds for perturbations of finite or large velocities.

EINAR HØILAND [4] discussed the horizontal motion in a rotating fluid. He got some exact stationary solutions of the vorticity equation of the type as shown in equation (10). CRAIG [5] and NEAMTAN [6] obtained more general solutions of the vorticity equation in its complete form without linearization.

The perturbation streamlines for $m=2$ are shown in Fig. 62 in a remarkable paper by HAURWITZ. The total velocity streamlines for $m=2$, $A_m=a^2\Omega/2400$, $B_m=0$, are shown in Fig. 2a as an example. The nodal parallels are at the equator and at 35.3° latitude. The streamlines are drawn in every $2a^2\Omega/240$, except those shown by broken curves. The distance between two streamlines is inversely proportional to the velocity of motion.

The streamlines surround the pole, and the effects of the disturbance appears as north-southerly undulations or meandering of the streamlines. The amplitude of this undulation depends on the constant A_m. The westerly current exhibits undulations with large amplitude over the hemisphere but does not split.

Fig. 2b corresponds to a relatively large value of the constant A_m, i. e., the total velocity streamlines for $m=2$, $A_m=a^2\Omega/600$, $B_m=0$ are shown in Fig. 2b. Here we have eight cells, i. e. two polar cyclones, two subtropical anticyclones, two tropical cyclones and two equatorial anticyclones. In each two adjoining cells the direction of the perturbation motion is opposite.

After REX [7*a*, 7*b*] and RIEHL [8], the term "blocking action" is understood to

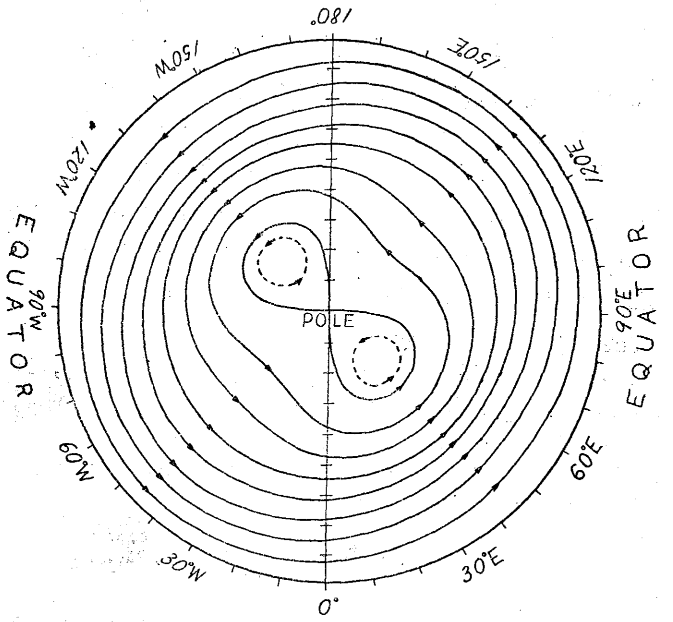

Fig. 2a The total stream-lines: $m=2$, $A_m=a^2\Omega/2400$, $B_m=0$.
Nodal lines: $\phi=0°$, $35°16'$

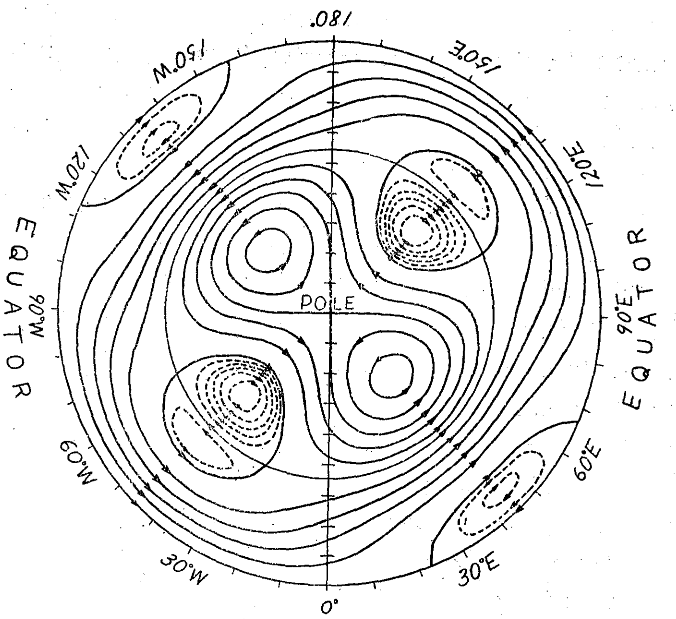

Fig. 2b The total stream-lines: $m=2$, $A_m=a^2\Omega/600$, $B_m=0$.
Nodal lines: $\phi=0°$, $35°16'$
Hyperbolic points: pole,

($\phi=35°16'$, $\lambda=112°48'$), ($\phi=0°$, $\lambda=22°48'$)
($\phi=35°16'$, $\lambda=157°12'$), ($\phi=0°$, $\lambda=67°12'$)
($\phi=35°16'$, $\lambda=292°48'$), ($\phi=0°$, $\lambda=202°48'$)
($\phi=35°16'$, $\lambda=337°12'$), ($\phi=0°$, $\lambda=247°12'$)

flow patterns observed in the middle troposphere (500 mb) that have the following characteristics:

(1) The basic westerly flow splits into two separate branches. Each of these branches transports appreciable mass——about half of the 500 mb contours go north and the other half south.

(2) The double-jet system extends over 45° of longitude.

(3) In well-developed blocks we observe a dynamic high just south of the northern current branch and a dynamic low just north of the southern branch. This pattern of *high in the north and low in the south* with easterly winds aloft in middle latitudes, is typical of blocks. A sharp transition from zonal type flow upstream to meridional type downstream is observed across the current split.

(4) The pattern persists with recognizable continuity for ten days, and can be considered as quasi-stationary flow.

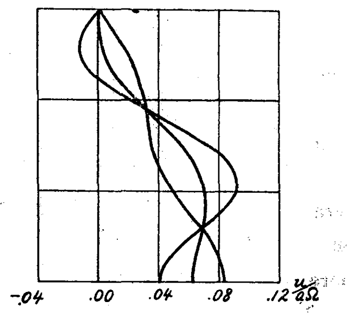

Fig. 3 (a).

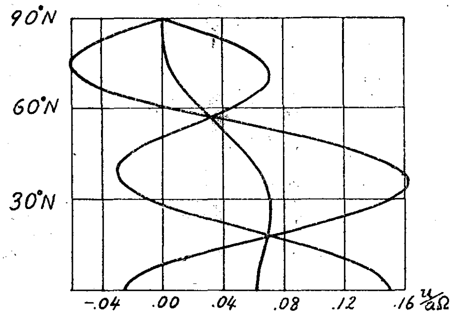

Fig. 3 (b).

Fig. 2b exhibits the following characteristics: (1) the basic westerly flow splits into two separate branches extending over 45° of longitude. (2) Each branch transports an appreciable mass. (3) A dynamic high occurs just south of the northern current branch and a dynamic low just north of the southern branch. (4) A sharp transition from zonal type flow upstream to meridional type downstream is observed across the current split. This may offer a simple explanation for the kinematical features of the well developed blocks in the middle troposphere.

The velocity profiles along selected meridians corresponding to Fig. 2a and Fig. 2b are shown in Fig. 3a and Fig. 3b, respectively.

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