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On the Coupling between Upper Waves and Surface Pressure Centers (II)

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Abstract

The effect of non-stationary motion and eddy viscosity on the surface pressure change and moving velocity of surface pressure center is estimated.

1. Introduction

The problem of pressure variation should be desirably dealt with by the integration of full equations, namely equations of motion, of continuity, of state and of polytropic change, and may be determined by the initial state. But as is well known the integration is very difficult and we can not but rest satisfied with the relations which contain some quantities as parameters. Although these relations are not complete in themselves, we sometimes get very convenient relations such as explain the phenomena physically. Tendency equation is one of them, as is widely applied in modern meteorology. The wind field contained in tendency equation as parameter should, of course, be the real wind field as a result of every relating effect, but is regarded as geostrophic or gradient wind for the first approximation to make up for the deficiency of observation.

As tendency equation gives no surface pressure change for geostrophic approximation neglecting the effect of curvature [1], the alternative equation which combines hydrostatic equation with that of polytropic change instead of continuity should be considered so as to estimate the effect of advection in baroclinic field. In the previous report [2] the effect of advection in the asymmetric temperature field was investigated by geostrophic approximation. This report is intended to combine the horizontal equation of motion with modified tendency equation in order to elevate the degree of approximation.

2. Combination of the equation of motion and the modified tendency equation

Let us denote polytropic potential temperature θ [3] which is assumed to be conserved in the free atmosphere, for example, potential temperature for adiabatic change, i.e.,

(1) $$\frac{d\theta}{dt}=0,$$

(2) $$\theta=T\left(\frac{p_0}{p}\right)^{\frac{\Gamma-1}{\Gamma}},$$

where Γ is politropic coefficient. The pressure gradient force may be written as follows,

(3) $$\frac{1}{\rho}\ \mathrm{grad}\ p=RT\ \mathrm{grad}\ (\ln p)=R\theta\left(\frac{p}{p_0}\right)^{\frac{\Gamma-1}{\Gamma}}\ \mathrm{grad}\ (\ln p)$$

$$=R\frac{\Gamma}{\Gamma-1}\ \theta\ \mathrm{grad}\ \left(\frac{p}{p_0}\right)^{\frac{\Gamma-1}{\Gamma}}$$

$$=\theta\ \mathrm{grad}\ \pi,$$

where $\pi=R\frac{\Gamma}{\Gamma-1}\left(\frac{p}{p_0}\right)^{\frac{\Gamma-1}{\Gamma}}$.

Using these quantities, the equations of motion are written as

(4) $$\frac{du}{dt}-fv=-\theta\frac{\partial\pi}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2},$$

(5) $$\frac{dv}{dt}+fu=-\theta\frac{\partial\pi}{\partial y}+\nu\frac{\partial^2 v}{\partial z^2},$$

(6) $$0=-g-\theta\frac{\partial\pi}{\partial z}.$$

The hydrostatic equation (6) is sufficient to estimate not the motion of air particle itself but pressure field. Putting (1) to the local time variation of (6), we get

(7) $$\frac{\partial^2\pi}{\partial z\partial t}=-g\frac{\partial\theta^{-1}}{\partial t}=g\left(u\frac{\partial\theta^{-1}}{\partial x}+v\frac{\partial\theta^{-1}}{\partial y}+w\frac{\partial\theta^{-1}}{\partial z}\right).$$

To investigate the pressure variation by horizontal advection of conservative temperature θ, we denote

$$u\frac{\partial\theta^{-1}}{\partial x}+v\frac{\partial\theta^{-1}}{\partial y}\equiv A,$$

$$u\frac{\partial\theta^{-1}}{\partial y}-v\frac{\partial\theta^{-1}}{\partial x}\equiv B,$$

and neglect the effect of temperature change by vertical motion or $\partial\theta^{-1}/\partial z=0$, then (7) becomes

(7′) $$\frac{\partial^2\pi}{\partial z\partial t}=gA.$$

If we multiply $\partial\theta^{-1}/\partial x$ or $\partial\theta^{-1}/\partial y$ on each of (4) and (5), and add or substitute them, we get the differential equation for A and B,

(8) $$\frac{dA}{dt}+fB-u\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial x}-v\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial y}=-\theta\left(\frac{\partial\pi}{\partial x}\frac{\partial\theta^{-1}}{dx}+\frac{\partial\pi}{\partial y}\frac{\partial\theta^{-1}}{\partial y}\right)+\nu\frac{\partial^2 A}{\partial z^2},$$

(9) $$\frac{dB}{dt}-fA-u\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial y}+v\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial x}=-\theta\left(\frac{\partial\pi}{\partial x}\frac{\partial\theta^{-1}}{\partial y}-\frac{\partial\pi}{\partial y}\frac{\partial\theta^{-1}}{\partial x}\right)+\nu\frac{\partial^2 B}{\partial z^2}.$$

Using equation (1) the following relations stand,

$$\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial x}=\frac{\partial}{\partial x}\frac{d\theta^{-1}}{dt}-\frac{\partial u}{\partial x}\frac{\partial\theta^{-1}}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial\theta^{-1}}{\partial y}-\frac{\partial w}{\partial x}\frac{\partial\theta^{-1}}{\partial z}$$

$$=-\frac{\partial u}{\partial x}\frac{\partial\theta^{-1}}{\partial x}-\frac{\partial v}{\partial x}\frac{\partial\theta^{-1}}{\partial y}\cong-\frac{\partial A}{\partial x},$$

$$\frac{d}{dt}\frac{\partial\theta^{-1}}{\partial y}=-\frac{\partial u}{\partial y}\frac{\partial\theta^{-1}}{\partial x}-\frac{\partial v}{\partial y}\frac{\partial\theta^{-1}}{\partial y}\cong-\frac{\partial A}{\partial y},$$

if the higher derivatives of the temperature field can be neglected. Therefore (8) and (9) become

$$(8')\qquad \frac{\partial A}{\partial t}+fB+2u\frac{\partial A}{\partial x}+2v\frac{\partial A}{\partial y}+w\frac{\partial A}{\partial z}=-\theta\left(\frac{\partial\pi}{\partial x}\frac{\partial\theta^{-1}}{\partial x}+\frac{\partial\pi}{\partial y}\frac{\partial\theta^{-1}}{\partial y}\right)+\nu\frac{\partial^2 A}{\partial z^2},$$

$$(9')\qquad \frac{\partial B}{\partial t}-fA+u\left(\frac{\partial B}{\partial x}+\frac{\partial A}{\partial y}\right)+v\left(\frac{\partial B}{\partial y}-\frac{\partial A}{\partial x}\right)+w\frac{\partial B}{\partial z}=-\theta\left(\frac{\partial\pi}{\partial x}\frac{\partial\theta^{-1}}{\partial y}-\frac{\partial\pi}{\partial y}\frac{\partial\theta^{-1}}{\partial x}\right)+\nu\frac{\partial^2 B}{\partial z^2}.$$

If we take natural coordinate s, n and $u=V\cos\alpha$, $v=V\sin\alpha$ where V is total velocity and α is the angle between x axis and stream line s, we arrive after several calculation at

$$2u\frac{\partial A}{\partial x}+2v\frac{\partial A}{\partial y}=\left(D+V\frac{d\alpha}{dn}\right)A+\left(\zeta+V\frac{d\alpha}{ds}\right)B+\frac{\partial\frac{V^2}{2}}{\partial x}\frac{\partial\theta^{-1}}{\partial x}+\frac{\partial\frac{V^2}{2}}{\partial y}\frac{\partial\theta^{-1}}{\partial y},$$

$$u\left(\frac{\partial B}{\partial x}+\frac{\partial A}{\partial y}\right)+v\left(\frac{\partial B}{\partial y}-\frac{\partial A}{\partial x}\right)=-V\frac{d\alpha}{dn}B-\left(\zeta+\frac{\partial V}{dn}\right)A+\frac{\partial\frac{V^2}{2}}{\partial x}\frac{\partial\theta^{-1}}{\partial y}-\frac{\partial\frac{V^2}{2}}{\partial y}\frac{\partial\theta^{-1}}{\partial x}.$$

Therefore (8′) and (9′) become

$$(8'')\qquad \frac{\partial A}{\partial t}+\left(D+V\frac{d\alpha}{dn}\right)A+\left(Z+V\frac{d\alpha}{ds}\right)B+w\frac{\partial A}{\partial z}=-\theta\nabla\pi'\cdot\nabla\theta^{-1}+\nu\frac{\partial^2 A}{\partial z^2},$$

$$(9'')\qquad \frac{\partial B}{\partial t}-V\frac{\partial\alpha}{dn}B-\left(Z+\frac{dV}{dn}\right)A+w\frac{\partial B}{dz}=-\theta\nabla\pi'\times\nabla\theta^{-1}\cdot\boldsymbol{k}+\nu\frac{\partial^2 B}{\partial z^2},$$

where D is horizontal divergence, Z absolute vorticity $\zeta+f$, ∇ the operation of horizontal gradient and π' is the function of dynamic pressure. If the present field quantity θ, π, D, Z, V, α, w are known, the above equations determine A which corresponds to pressure tendency. The equation obtained by neglecting all the terms except the third term on the left hand and the first term on the right hand side in (9″), is nothing but the modified tendency equation by geostrophic approximation which was discussed in the former report.

3. Approximate expression for surface pressure change

Non-geostrophic effect of non-stationary curvilinear wind field and friction may be calculated from equations (8″) and (9″). In this section we intend to estimate it formally by successive approximation. Substituting B given by (9″) into (8″), we get

$$(10)\qquad \left\{Z+\frac{dV}{dn}-\frac{V\frac{d\alpha}{dn}\left(D+V\frac{d\alpha}{dn}\right)}{Z+V\frac{d\alpha}{ds}}+\frac{d'}{dt}\left(\frac{D+V\frac{d\alpha}{dn}}{Z+V\frac{d\alpha}{ds}}\right)-\nu\frac{\partial^2}{\partial z^2}\left(\frac{D+V\frac{d\alpha}{dn}}{Z+V\frac{d\alpha}{ds}}\right)\right\}A$$

$$=\theta\nabla\pi'\times\nabla\theta^{-1}\cdot\boldsymbol{k}+\frac{V\frac{d\alpha}{dn}}{Z+V\frac{d\alpha}{ds}}\theta\nabla\pi'\cdot\nabla\theta^{-1}-w\frac{\partial}{\partial z}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)+\nu\frac{\partial^2}{\partial z^2}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)$$

$$-\frac{\partial}{\partial t}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)-\left[\frac{D}{Z+V\frac{d\alpha}{ds}}+\frac{d'}{dt}\left(\frac{1}{Z+V\frac{d\alpha}{ds}}\right)-\nu\frac{\partial^2}{\partial z^2}\left(\frac{1}{Z+V\frac{d\alpha}{ds}}\right)\right]\frac{\partial A}{dt}$$

$$-\frac{1}{Z+V\frac{d\alpha}{ds}}\frac{\partial^2 A}{\partial t^2}+F\left(\frac{\partial A}{\partial z}\right)+O\left(\frac{\partial^2\theta^{-1}}{\partial x^2}\right).$$

In the right hand side of this equation, the first term expresses the effect of horizontal advection along isobars, the second that across isobars, accompanied by divergence, the third vertical motion, the fourth friction and the fifth and the sixth non-stationary motion respectively for the pressure tendency. The last two terms are higher order quantity, and therefore are not considered here. By successive approximation we get

$$(10')\qquad \left(Z+\frac{dV}{dn}\right)A=\theta\nabla\pi'\times\nabla\theta^{-1}\cdot\boldsymbol{k}+\frac{V\frac{d\alpha}{dn}}{Z+V\frac{d\alpha}{ds}}\theta\nabla\pi'\cdot\nabla\theta^{-1}-w\frac{\partial}{\partial z}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)$$

$$+\nu\frac{\partial^2}{\partial z^2}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)-\frac{\partial}{\partial t}\left(\frac{\theta\nabla\pi'\cdot\nabla\theta^{-1}}{Z+V\frac{d\alpha}{ds}}\right)$$

$$-\frac{D}{Z+V\frac{d\alpha}{ds}}\frac{\partial}{\partial t}\left\{\frac{\theta\nabla\pi'\times\nabla\theta^{-1}\cdot\boldsymbol{k}}{Z+\frac{dV}{dn}}+\frac{V\frac{d\alpha}{dn}\theta\nabla\pi'\cdot\nabla\theta^{-1}}{\left(Z+V\frac{d\alpha}{ds}\right)\left(Z+\frac{dV}{dn}\right)}\right\},$$

where higher order quantities and small quantities in the coefficients of each terms are all neglected.

Integrating (10′) vertically from surface s to level H we get

$$(11)\qquad -\frac{1}{g}\overline{Z+\frac{dV}{dn}}\frac{\partial\pi_s}{\partial t}=-\frac{1}{g}\overline{Z+\frac{dV}{dn}}\frac{\partial\pi_H}{\partial t}+H\theta\nabla\pi_s'\times\nabla\theta^{-1}\cdot\boldsymbol{k}+\frac{\overline{w}}{Z+V\frac{d\alpha}{ds}}gH\theta(\nabla\theta^{-1})^2$$

$$-\nu\frac{\partial}{\partial z}\left(\frac{1}{Z+V\frac{d\alpha}{ds}}\right)_s\theta\nabla\pi_s'\cdot\nabla\theta^{-1}-\frac{1}{Z+V\frac{d\alpha}{ds}}\frac{\partial}{\partial t}\left[H\theta\nabla\pi_s'\cdot\nabla\theta^{-1}-\frac{H}{2}gH\theta(\nabla\theta^{-1})^2\right]$$

$$-\frac{\overline{D}}{Z+V\frac{d\alpha}{ds}}\frac{\partial}{\partial t}\left(\frac{H\theta\nabla\pi_s'\times\nabla\theta^{-1}\cdot\boldsymbol{k}}{Z+\frac{dV}{dn}}\right),$$

where we use the pressure field on the right hand side $\theta\nabla\pi'=\theta\nabla\pi_s'-g\theta z\nabla\theta^{-1}$ assuming θ to be constant vertically, and neglect the trivial terms. From the above it can be concluded that the followings have effect on surface pressure fall, i.e.,

1) pressure fall in upper layer (the first term),
2) the geostrophic advection of warm air (the second term),
3) upward motion (the third term),

4) inflow of warm air towards low pressure by surface friction (the fourth term),
5) increase of temperature gradient (the fifth term),
6) convergence and increasing warm air advection or divergence and decreasing warm air advection (the sixth term).

As to the stagnant surface centers, deepening is caused especially by 1), 3) and 5).

4. Moving velocity of surface pressure centers

From equation (11) the moving velocity of symmetric surface pressure center $\boldsymbol{c}$ is estimated by kinematic method [4], as in the previous report. We finally obtain the equation

$$(12)\qquad \boldsymbol{c}=C\frac{\frac{\partial}{\partial x}\left(\theta\frac{\partial \pi_H}{\partial x}\right)}{\frac{\partial}{\partial x}\left(\theta\frac{\partial \pi_s}{\partial x}\right)}\boldsymbol{i} + \frac{gH}{Z+\frac{dV}{dn}}\left[\boldsymbol{k}\times\nabla \ln\theta+\frac{1}{Z+V\frac{d\alpha}{ds}}\left\{\frac{\partial}{\partial t}\nabla \ln\theta+\frac{D}{Z+\frac{dV}{dn}}\frac{\partial}{\partial t}\nabla \ln\theta\times\boldsymbol{k}\right\}\right.$$
$$\left. -\frac{1}{\left(Z+\frac{dV}{dn}\right)\left(Z+V\frac{d\alpha}{ds}\right)}\left\{\frac{D}{Z+\frac{dV}{dn}}\frac{\partial^2}{\partial t^2}\nabla \ln\theta+\boldsymbol{k}\times\frac{\partial^2}{\partial t^2}\nabla \ln\theta\right\}\right],$$

where C is the velocity of upper wave. The first term shows the steering by upper wave and second term shows that cyclones travel along the isotherms facing warm area to the right and the following terms show the effect of non-stationary motion. Above all the third term may be written as $\frac{g_H}{Z+\frac{dV}{dn}}\,\mathrm{grad}\,\frac{\Delta \ln T}{\Delta t}$ which shows that the cyclone moves towards warming center.

References

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