A Method of Measuring the Ion Spectrum

by

M. Misaki

Magnetic Observatory (Kakioka)

(Communicated by H. Hatakeyama)

(Received September 16, 1950)

Abstract

In measuring the mobility-spectrum of the ions in the atmosphere, by means of the cylindrical condenser of the type of Gerdien's aspiration apparatus, we must calculate the second differential coefficient of the voltage-current characteristic curve. But it is very difficult to derive this from an experimental curve. Furthermore, the variation of ion contents during the observation can not be neglected without causing appreciable error to the result.

I used Nolan's "Sub-divided Condenser", and devised a method in which we have only to derive the first differential coefficient for this purpose.

1. Introduction

Methods of measuring the mobility-spectrum of ions in the atmosphere have been devised by H. Israel,(1) P. J. Nolan,(2) J. Booij(3) and others. The difficulties which lie in ionic spectrum measurement are due to variation during the measurement or the actual treatment which will be mentioned below. And then, the above-mentioned investigators have devised various methods—each of them has its own valuable specialities. For instance, H. Israel has devised his "Double Condenser Methed" intending to control the variation of the total ion number during the observation. In this method, however, a high degree of exactitude in observation is required because of the fact that to obtain the ion spectrum one must derive the second differencial quotients of the current-potential characteristic curve.

Though the 2nd derivative is still required in P. J. Nolan's "Subdivided Condenser Method", it has the advantage that his current-potential characteristic shows distinctly the positions of the predominant ion groups in a spectrum.

I have devised a rather simple method to measure the fine structure of a spectrum. It seems also to be useful for the measurement of conductivity.

2. The principle of measurement

In the first place we consider the Gerdien's type aspiration apparatus as shown in Fig. 1. The potential applied to the outer cylinder is V, and the inner cylinder is earthed. The air which flows between them is Φ per sec.. In these conditions, the mobility k_c of ion, whose trajectory is just from point A to point B, is

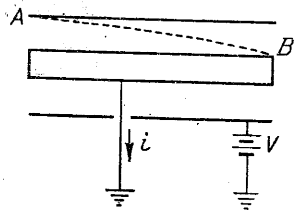

Fig. 1.

$$k_c=\frac{\Phi}{4\pi CV},$$

where C is the capacity of the cylindrical condenser.

The numerical value of the element dn of a mobility-region between k and $k+dk$ is found from the equation $dn=f(k)dk$.

The ions whose mobilities are larger than k_c constitute saturation current

$$i'=\Phi e\int_{kc}^{\infty}f(k)dk,$$

and the ions, whose mobilities are less than k_c, are not all caught in the inner cylinder, that is, the number of ions which are caught is $\frac{k}{k_c}$ times of the total number of ions whose mobility is k. So that the current consisting of these ions is

$$i''=\Phi e\frac{1}{k_c}\int_0^{kc}kf(k)dk.$$

The current between the inner and the outer cylinder is $i'+i''$, and

$$i=i'+i''=\Phi e\int_{kc}^{\infty}f(k)dk+\Phi e\frac{1}{k_c}\int_0^{kc}kf(k)dk,$$

then,

$$\frac{d^2i}{dV^2}=\frac{d}{dk_c}\left(\frac{di}{dk_c}\frac{dk_c}{dV}\right)\frac{dk_c}{dV}=-\frac{e\Phi^2}{4\pi c}\frac{1}{V^3}f(k_c).$$

Thus, the second derivative of the current-potential characteristic gives the distribution of ions among the individual mobilities. But to obtain the 2nd derivative from the experimental curve is very difficult, and I devised a method in which we have only to derive the first derivative.

I used Nolan's Subdivided Condenser (Fig. 2). The inner cylinder of the condenser is divided at point B, and each part is earthed. The outer cylinder is charged at potential V.

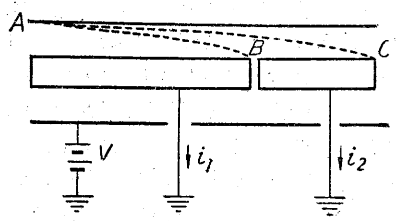

Fig. 2.

The current i_1 that flows from the first part of the inner cylinder to the earth is

(1) $$i_1=\Phi e\left[\int_{kc}^{\infty}f(k)dk+\frac{1}{k_c}\int_0^{kc}kf(k)dk\right],$$

where k_c is the mobility of the ion which enters from point A and reaches point B, and is given as

(2) $$k_c=\frac{\Phi}{4\pi C_1 V},$$ C_1 : capacity of the 1st condenser.

Similarly the mobility of the ion which enters from A and reaches C is

(3) $$k_a=\frac{\Phi}{4\pi(C_1+C_2)V},$$ C_2 : capacity of the 2nd condenser.

In the second condenser, all ions whose mobilities are larger than k_a and escaped from the lst condenser are caught. The current constituted of these ions is

$$i_2'=\Phi e\int_{ka}^{kc}(1-\frac{k}{k_c})f(k)dk,$$

whereas the ions whose mobilities are less than k_a form a current independent of the existence of lst condenser

$$i_2''=\Phi e\left(\frac{1}{k_a}-\frac{1}{k_c}\right)\int_0^{ka}kf(k)dk.$$

And so the current i_2 flowing from the second inner cylinder to the earth is

(4) $$i_2=i_2'+i_2''=\Phi e\left[\int_{ka}^{kc}f(k)dk+\frac{1}{k_a}\int_0^{ka}kf(k)dk-\frac{1}{k_c}\int_0^{kc}kf(k)dk\right].$$

Consequently we obtain the following equation from (1) and (4),

(5) $$\frac{C_2}{4\pi e(C_1+C_2)}\frac{d}{dV}\left(\frac{i_1}{C_1}-\frac{i_2}{C_2}\right)=\int_{ka}^{kc}kf(k)dk=\bar{k}\int_{ka}^{kc}f(k)dk$$

where

$$\bar{k}=\int_{ka}^{kc}kf(k)dk\bigg/\int_{ka}^{kc}f(k)dk\doteqdot\frac{1}{2}(k_c+k_a).$$

Fquation (5) gives $\int_{ka}^{kc}f(k)dk$ instead of $f(k)$, but with this we can obtain a practically sufficient ionic spectrum.

The relation between k_c and k_a is given from (2) and (3) as follows

$$k_a=\frac{C_1}{C_1+C_2}k_c.$$

The smaller C_2 is, the finer becomes the structure of the spectrum.

Thus, it is not necessary to obtain the second derivative but sufficient to derive the first. And moreover the result is hardly affected by variation of ion content during the measurment by the aid of a suitable amplifying apparatus.

For measuring $\frac{i_1}{C_1}-\frac{i_2}{C_2}$ without an amplifier, a quadrant-electrometer is convenient. In such a case, the lst and the 2nd inner cylinders are to be connected with two pairs of quadrants of electrometer of the needle which is charged at a constant high potential v_3, and we may observe the deflection angle of the needle. We introduce the expressions v_1 and v_2 for the potential of the lst and the 2nd inner cylinder, then we get the following relation at the constant potential of the outer cylinder

$$\frac{i_1}{C_1}-\frac{i_2}{C_2}=\left(1+\frac{C_1'}{C_1}\right)\left\{\frac{dv_1}{dt}+\alpha v_1\right\}-\left(1+\frac{C_2'}{C_2}\right)\left\{\frac{dv_2}{dt}+\alpha v_2\right\},$$

where C_1' denotes the capacity of quadrant and lead wire which are connected to the lst inner cylinder, and similarly C_2' with respect to the 2nd cylinder system, α

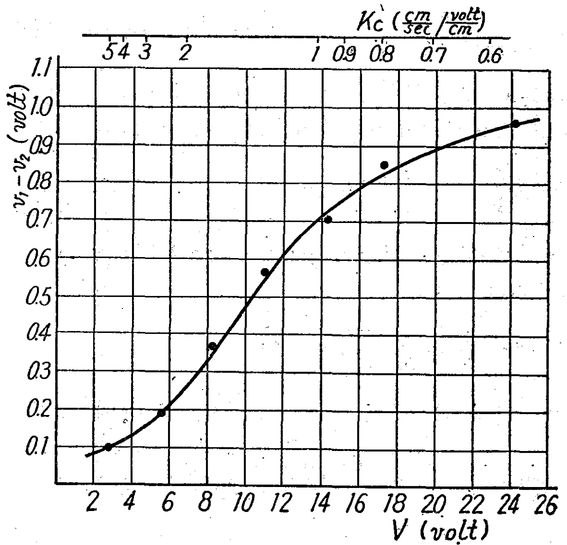

Fig. 3.

is determined from the preliminary observation of natural leakage. Let $v_1=v_2=0$ at $t=0$, then

$$\left(1+\frac{C_1'}{C_1}\right)v_1-\left(1+\frac{C_2'}{C_2}\right)v_2=(1-e^{-\alpha t})\frac{1}{\alpha}\left(\frac{i_1}{C_1}-\frac{i_2}{C_2}\right).$$

If we make $\frac{C_1'}{C_1}=\frac{C_2'}{C_2}=p$, adjusting C_1' (adding suitable capacity to the lst system), then we get

$$\frac{i_1}{C_1}-\frac{i_2}{C_2}=(1+p)\frac{\alpha}{1-e^{-\alpha t}}(v_1-v_2),$$

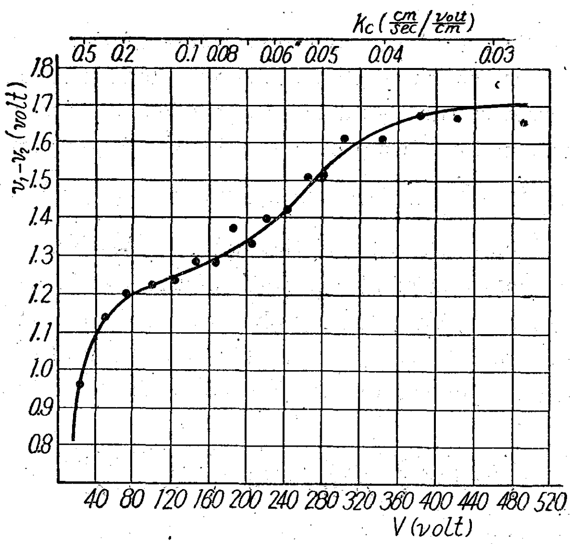

Fig. 4.

where v_1-v_2 is proportional to the deflecting angle of the needle, and we can calculate the numerical value of $\frac{i_1}{C_1}-\frac{i_2}{C_2}$. From thus formed V, $\left(\frac{i_1}{C_1}-\frac{i_2}{C_2}\right)$ characteristic curve ion-spectrum can be obtained.

3. An example of the observation

In this section is shown an example of observation carried out with this method. The dimensions of the measuring apparatus are as follows;

radius of the inner cylinder ......0.5 cm
radius of the outer cylinder......2.5 cm
length of the 1st inner cylinder ......60 cm
length of the 2nd inner cylinder......20 cm
Φ......320 c. c./sec
C_1 ......18.6 cm
C_2......6.2 cm
C_1'......12.5 cm
C_2' ......4.2 cm

In this experiment, a Dorezalek quadrant electrometer was used. In Figs. 3, 4, the values of v_1-v_2 3 minutes after the beginning of observation, instead of $\frac{i_1}{C_1}-\frac{i_2}{C_2}$, are taken as the ordinate.

The ion spectrum derived from Figs. 3, 4 is shown in Fig. 5. The two curves, C and D, in this diagram represent k_c, n-curve and k_d, n-curve respectively, so that if the abscissae of the intersection points of a line $n=n_1$ and C-, D-curves are k_{c1} and k_{d1}, n_1 is the number of ions whose mobilities are between k_{c1} and k_{d1}. Inversely the number of ions in any mobility region is given by dividing the interval of two

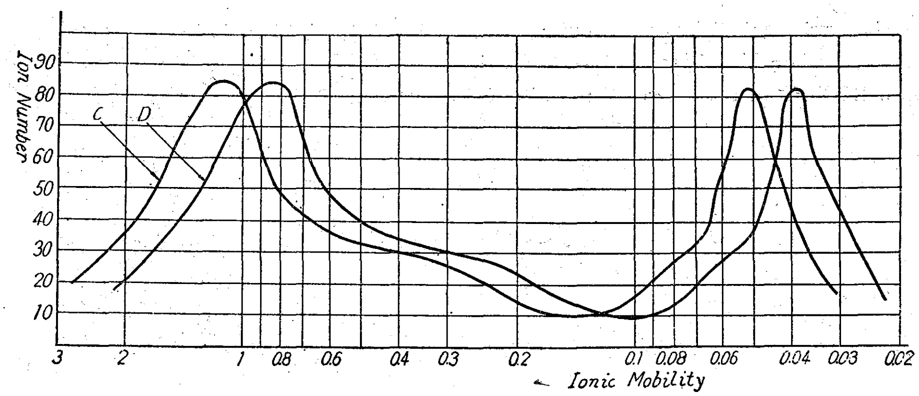

Fig. 5.

curves "stairwise" as in Fig. 6, and summing up the ordinates of the intersection points on the C-curve. From Fig. 5, we see that about 350 ions are found in the region 2.5~0.5 $\left(\frac{cm}{sec} \middle| \frac{volt}{cm}\right)$ and 200 ions in the region 0.1~0.02 $\left(\frac{cm}{sec} \middle| \frac{volt}{cm}\right)$.

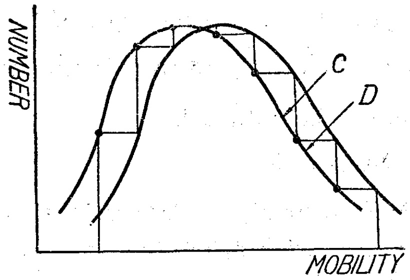

Fig. 6.

4. Summary

In this paper a method of measuring ion-spectrum is described. Its specialities are as follows:

(1) The inner cylinder of aspiration apparatus is divided in two parts. We observe the value $\left(\frac{i_1}{C_1}-\frac{i_2}{C_2}\right)$, and form the characteristic curve, taking $\left(\frac{i_1}{C_1}-\frac{i_2}{C_2}\right)$ as ordinate and potential applied to the outer cylinder as abscissa. (i_1, C_1, i_2, C_2 are current and capacity of each part respectively.)

(2) Ion-spectrum is obtained by deriving the lst derivative of its characteristic, and the 2nd derivative is not needed.

(3) For trial, I observed by this method, using a quadrant-electrometer. An example of this observation is shown in Figs. 3~5.

In conclusion I wish to thank Dr. H. Hatakeyama and Dr. S. Imamiti for their kind guidance and encouragement throughout the work.

References

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