On the Intensities and Altitudes of the Night-Sky Light*

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Abstract

Taking into consideration of the atomospheric extinction, refraction, and scattering, the formulae for the received intensities of the night-sky light into a solid angle was introduced geometrically, where the luminescent layer assumes a geoconcentric spherical surface of the uniform radiant intensity. An attempt was made to evaluate the height of the layer from the observed data with the aid of these formulae. Finally, discussing the general assumptions, the authors propose possible geographic distributions of the radiant intensities and the simultaneous observation to which the formulae are applicable.

1. Introduction and General Assumptions

Comparing the observed intensities of the night-sky light from the zenith and the horizon, the altitude of the luminescent layer of the night-sky light has been estimated by several authors.(1) The principle of their estimations is baced upon the following fundamental assumptions:

(1) Disregarding the thickness of the layer, an average level may be only taken into consideration.

(2) The layer assumes a spherical surface, which is concentric with the earth.

(3) The radiant intensities may be distributed uniformly over the whole surface of the layer.

The estimated values of the altitude of the layer, howevere, have been different one another within the range of 50~400 km above the sea level. To investigate the nature of these discrepancies and the propriety of the general assumptions, the authors attempted to find whether or not the two values, which have been obtained from the two ratios of the intensities of the monochromatic light at three zenith distances, agree with each other.

2. Formulae for the Intensities of the Incident Light

In the unit area of the layer there are N particles, each of which radiates energy ε. If ε and N are independent quantities of zenith distances, the radiant

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intensities $E=\varepsilon N$ are disrtibuted uniformly. But the incident intencities, which come to a point on the earth's surface, have suffered divergency and atmospheric extinction before thier arrival, so they are functional quantities of zenith distnces.

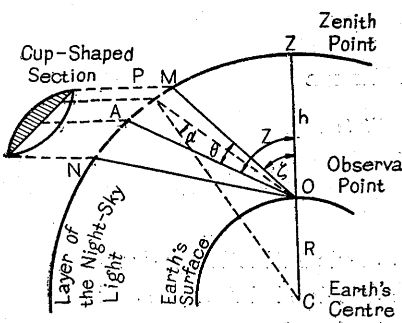

Fig. 1

In Fig. 1 the zenith angle of the axial direction and the angular aperture of the condenser cone are $\angle ZOA=z$ and $\angle MON=2\theta$ respectively. The section of the cone by the layer, which is cup-shaped, is divided into elementary bands according to the zenith distances ζ. After geometrical consideration the arc length l is found in the form of the expression

$$l=2\sin\zeta\cos^{-1}\left(\frac{\cos\theta-\cos z\cos\zeta}{\sin z\sin\zeta}\right),$$

then the elementary solid angle and area are respectively

$$d\Omega=l\,d\zeta \quad \text{and} \quad dA=r^2\sec\alpha\, d\Omega,$$

where r is the linear distance $\overline{OP}$ of an elementaly band and α is its geocentric parallax, and further the facter $\sec\alpha$ has the well-known expression in terms of the earth's radius $\overline{CO}=R$ and the altitude of the layer above the earth's surface $\overline{OZ}=h$, namely

$$\sec\alpha=(R+h)\Big/\sqrt{(R+h)^2-(R\sin\zeta)^2}.$$

The luminescent energy $E\,dA$ radiated from an elementary band is reduced to $E\,dA/(4\pi r^2)$ due to divergency and then to $\frac{p\tau E\,dA}{4\pi r^2}$ due to atmospheric extinction, where p is the transmission coefficient in the zenith and τ is the reducing fraction from the zenith to a certain zenith distance generally expressed in the form [(2)] $\tau=p^{F(\zeta)-1}$ and

$$F(\zeta)=j_0\sec\zeta-\frac{j_1}{R}\sec\zeta\tan^2\zeta+\frac{3}{2}\frac{j_2}{R^2}\sec^3\zeta\tan\zeta-\cdots\cdots.$$

The intensity of incident energy dI which comes from an elementary band, therefore, becomes

$$dI=p\tau E\,dA\Big/(4\pi r^2),$$

and these dI should be integrated over all the bands in the cup-shaped section, if the intensity of the whole incident energy I which is received directly by the condenser corn is to be obtained. We may call I merely the incident intensity and can obtatn its formulae as follows: in the case of $z\geqq\theta$,

$$I=\frac{pE}{2\pi}\int_{z-\theta}^{z+\theta}\frac{\tau(R+h)\sin\zeta}{\sqrt{(R+h)^2-(R\sin\zeta)^2}}\cdot\cos^{-1}\left(\frac{\cos\theta-\cos z\cos\zeta}{\sin z\sin\zeta}\right)d\zeta,$$

and in the case of $z=0$, similarly

$$I_0=\frac{pE_0}{2}\int_0^{\theta}\frac{\tau(R+h)\sin\zeta}{\sqrt{(R+h)^2-(R\sin\zeta)^2}}d\zeta.$$

3. Effects of the Astronomical Refraction

One of the effects of the astronomical refraction concerns the atmospheric extinction, and the general formula for the extinction is expressed taking the refraction into consideration.

Another part has a rôle to distort the condenser cone, accordingly the boundary of the cup-shaped section suffers the transformation. While the lights of the stars come from an almost infinite distance, the rays of the night-sky start from the points which are the intersections of the rays with the layer and whose distances are finite. Say the astronomical refraction angle of a star, whose true zenith distance is ζ, is η, its refracted ray intersects the layer at a point, whose apparent zenith distance is $\zeta+k\eta$, where k should be the complicated function of ζ and $1>k>0$. Taking this into account angular quantities in I should be transformed, say $\zeta\rightarrow\zeta+k\eta$ and the circular boundary of the cup-shaped section should be distorted into an oval or egg-shape.

Compairing the angular accuracies in the present observations, the effects of the astronomical refractions, which dose not exceed $3'$, may be negligible at any rate in our calcurations.

4. Formulae for the Intensities of the Scattered Light

The observed intencities of the night-sky light comprise the scattered intencities as well as the incident intencities, then we attempt to find the formulae for the scattered intersities without regard to the refraction and the second scattering. In Fig. 2 referring to the scattering point D the linear distance, the zenith distance and azimuth of the radiating point A are denoted, s, ξ and ϕ respectively, *i. e.* $s=\overline{AD}$ and $\xi=\angle YDA$. At A elementary erea on the layer, which is cut into section by $d\xi$ and $d\phi$, becomes

$$dC=s^2\,d\xi\,d\phi\Big/\sqrt{1-\left(\frac{R+x}{R+h}\sin\xi\right)^2},$$

where R is earth's radius, h and x are respectively the altitudes of the points A and D above the earth's surface.

EdC, the radiant energy of dC, is fractionized into

$$EdC.\frac{e^{-a}}{4\pi s^2},$$

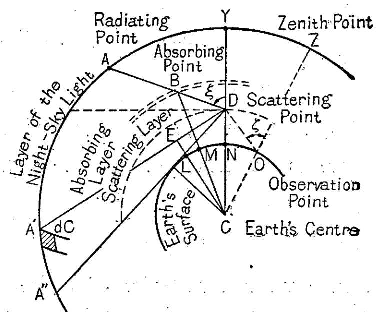

Fig. 2

when it gets to D. The factor e^{-a} is derived from the extinction, and if $0\leqq\xi\leqq 90°$, then

$$a=\kappa\int_{\overline{AD}}\sigma\,ds=\kappa\int_x^h\frac{\sigma\,dy}{\sqrt{1-\left(\frac{R+x}{R+y}\sin\xi\right)^2}}$$

and if

$$90°<\xi\leqq\xi_{max}=\angle YDA''=\sin^{-1}\left(\frac{R}{R+x}\right),$$

then $$a=\kappa\int_{\overline{A'D}}\sigma\, ds=2\kappa\int_{x_0}^{x} ,,\ dy+\kappa\int_{x}^{h} ,,\ dy.$$

In this expression, κ is the so-called absorption coeficient, σ the density of a certain absorbing layer B, y the altitude at B, and $x_0=\overline{EL}$ the lowest altitude on the line $\overline{A'D}$, namely

$$x_0=(R+x)\sin\xi-R.$$

At D each molecule scatters the incident lights which come from A, and sends the scattered lights to the observation point O. According to the Rayleigh's law the intensity ratio δ is calculated with the formula

$$\delta=\frac{\pi^2 T^2}{t^2\lambda^4}\left(\frac{\in-\in_0}{\in}\right)^2\frac{1+\cos^2\psi}{2}\,\frac{6+3\rho}{6-7\rho},$$

where π, λ, T, $\in$, $\in_0$, and ρ are the well-known notations, and t is the linear distance between D and O, namely the positive root of the equation

$$(R+x)^2=R^2+t^2+2Rt\cos\zeta,$$

and further ψ is the angle between $\overline{AD}$ and $\overline{DO}$, which we should be able to find in space geometry taking the earth's centre C as the origin and $\overline{CD}$ as the vertical axis.

An unit volume of the scattering layer contains n molecules of a certain kind, then there are ndV molecules of it in the elementary volume dV at D, and

$$dV=t^2\,d\Omega\,dx\Big/\sqrt{1-\left(\frac{R}{R+x}\sin\zeta\right)},$$

where the elementary solid angle $d\Omega$ is the same as in the section 2.

The scattered lights which start from D also suffer the atmospheric extinction on the way to O fractionized by the factor e^{-b}, where

$$b=\kappa\int_{\overline{DO}}\sigma\, dt=\kappa\int_0^x\frac{\sigma\, dx}{\sqrt{1+\left(\frac{R}{R+x}\sin\zeta\right)^2}}.$$

As a whole the intensity of the scattered light S which comes into the condenser cone is fromulated by the integration, whose integrand is

$$\frac{En}{4\pi s^2}e^{-(a+b)}\,\delta.\,dC\,dV,$$

and integration intervals of the variables are $[0,\ 2\pi]$, $[0,\ \xi_{\max}]$, $[0,\ h]$ and $[z-\theta,\ z+\theta]$ as to ϕ, ξ, x and ζ respectively. Summation as to T, $\in$, ρ and n should be finally calculated according to the kinds of molecules which compose the earth's atmosphere.

5. Observational Data and Evaluation of the Altitude

The observed intensity J should be the sum of the incident intensity I and the scattered intensity S. Given the values of θ, z, τ and h, we can evaluate the integration part H in the formula of I, while on the other hand the fourfold integral of S may require extremely laborious calculations to evaluate. Then we put

$$I \equiv p\,E\,H, \qquad I_0 \equiv pE_0\,H_0,$$

and approximately

$$S \equiv qE_0\,H_0, \qquad S_0 \equiv q_0\,E_0\,H_0,$$

where the suffixes refer to zenith distances. To eliminate the radiant intensities E which have not been observed directly, we take the ratios

$$\frac{I}{I_0}=\frac{EH}{E_0H_0} \quad \text{and} \quad \frac{J}{J_0}=\frac{pEH+q\,E_0\,H_0}{pE_0\,H^0+q_0\,E_0H_0},$$

I/I_0 is accordingly reduced from J/J_0, *i.e.*

$$\frac{I}{I_0}=\left(1+\frac{q_0}{p}\right)\frac{J}{J_0}-\frac{q}{p}.$$

We are indebted to Dr. M. Huruhata at Department of Astronomy, University of Tokyo for the observational data, which are

$\theta=11°$, $z=0°$, $54°\ 17'$ and $70°$,

$\lambda=5577\text{Å}$ (the auroral line of oxygen atoms).

For the light of this wave length we may adopt the extrapolated value by the observations at Mt. Wilson Observatory,[3] *i. e.* $p=0.742$, and we dare take the roughly interpolated values of qs by Dufay's[4] and Gauzit's[5] calculations as follows: $q_0=0.036$, $q=0.096$ and 0.107 for $\zeta=0°$, $54°\ 17'$ and $70°$ respectively.

Values of J_0, J_{54} and J_{70} observed by Huruhata have been reduced into I_{54}/I_0 and I_{70}/I_0, and plotted in Fig. 3. In the period from June 9. to July 8. 1948 the mean values have been

$$I_{54}/I_0=1.34 \text{ and } I_{70}/I_0=1.51.$$

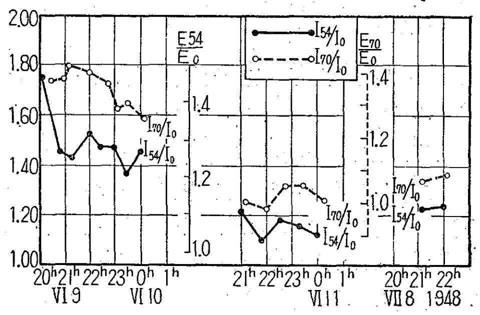

Fig. 3

On the other hand using the table of $F(\zeta)$, the theoritical values of Hs are introduced by numerical integrations of the fomulae, and the caluculated values of H_{54}/H_0 and H_{70}/H_0 are plotted against the assumed values of the altitude of the layer in Fig. 4.

If the distributions of the radiant intensities E are uniform over the layer, *i.e.* $E_{70}=E_{54}=E_0$ (Assumption(3)), then it becomes

$$I_{54}/I_0=H_{54}/H_0 \quad \text{and} \quad I_{70}/I_0=H_{70}/H_0,$$

and if the layer assumes a geo-concentric spherical surface (Assumption (2)), then I_{54}/I_0 and I_{70}/I_0 should simultaneously find the same value of h in Fig. 4.

The estimated values of h, however, have been

$$h=80\text{ m} \quad \text{for} \quad I_{54}/I_0=1.34$$
$$\text{and} \quad h=20\text{ m} \quad \text{for} \quad I_{70}/I_0=1.51,$$

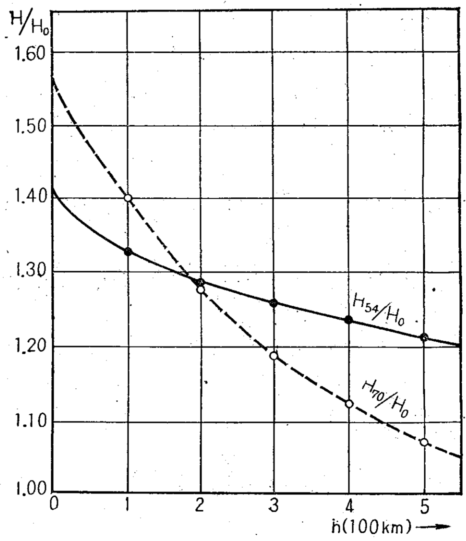

Fig. 4

furthermore the occasional values of I/I_0 sometimes lead to absurd values of h, say $h \leqq 0$.

It seems to us that these discrepancies and self-contradictions are due to the fundamental assumptions.

6. Geographical Distribution and Hourly Variation of the Radiant Intensities

Setting aside the assumption (3) but not (2) and (1), we are able to estimate the geographical distributions of the radiant intensities E in the cases of any altitudes h.

When a certain altitude of the layer is taken, the corresponding ratios H_{54}/H_0 and H_{70}/H_0 are found in Fig. 4, then $E_{54}/E_0=(I_{54}/I_0)(H_0/H_{54})$ and $E_{70}/E_0=(I_{70}/I_0)(H_0/H_{70})$ are estimatd. When the observed directions are all in the meridian, the geocentric latitude φ_ζ of the observed point P in the Fig. 1 is calculated by the equation

$$\angle \mathrm{ZCP}=\varphi_\zeta-\varphi_0=\zeta-a=\zeta-\sin^{-1}\frac{R\sin\zeta}{R+h},$$

where $\varphi_0=35°43'$ is the geocentric latitude of the observation point O, and the linear distance of the observed point from the zenith Z is

$$\widehat{\mathrm{ZP}}=(R+h)(\zeta-a).$$

Thus in Fig. 5 we may show the geographical distibutions of E/E_0 according to the several assumed h.

Hourly variations of E/E_0 are similar to those of I/I_0 provided that the scale values are changed in the relation of $\frac{E}{E_0}=\frac{H_0}{H}\frac{I}{I_0}$. So we may use the curve of as the curve of E/E_0 inserting the proper scales.

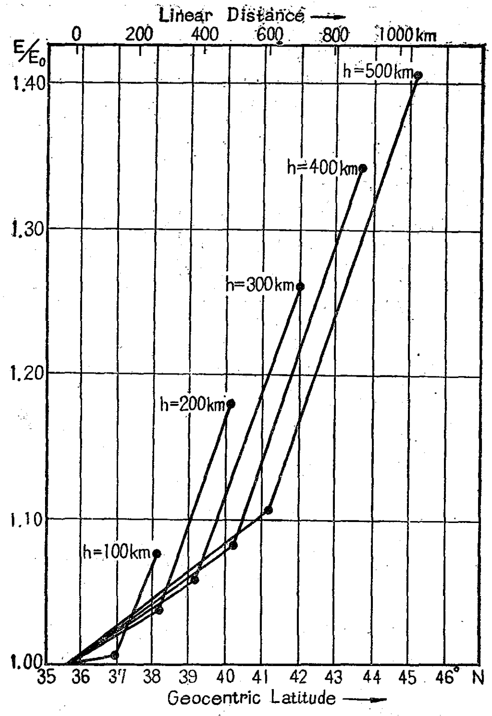

Fig. 5

7. Discussion and Acknowledgments

Another estimating method or the direct observation on distributions of E/E_0 is expected to decide the altitude of the layer, though it might vary from day to day.

If the fundamental assumption (2) besides (3) breaks down, we propose that a certain point on the layer should be observed simultaneously at several positions on the earth's surface. In this case it may not be meaningless to notice that first the observed point should be strictly identified from the three observation places or more, as the luminescent layer is an extended object, second the aperture of the condenser cone should not be too large, as the layer may be bent. On these procedures the formulae introduced above may become more valib.

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