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Preconditioned Optimizing Utility for Large-dimensional analyses (POpULar) in the MRI Multivariate Ocean Variational Estimation (MOVE) System

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1. Introduction

The MRI Multivariate Ocean Variational Estimation (MOVE) System is an ocean data assimilation system developed in Japan Meteorological Agency (JMA)/ Meteorological Research Institute (MRI). The analysis scheme in MOVE System is a variational (3DVAR/4DVAR) method with vertical coupled temperature-salinity EOF modes. Preconditioned Optimizing Utility for Large-dimensional Analyses (POpULar) is, then, applied for minimizing a nonlinear cost function which includes inversion of a non-diagonal background error covariance matrix with the horizontal error correlations. The meaning of applying POpULar is presented in this poster.

2. MOVE System

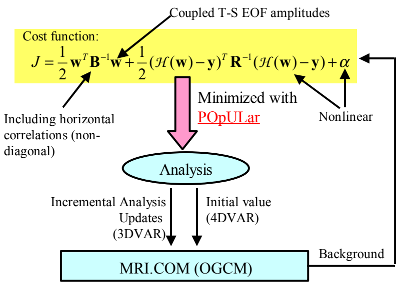


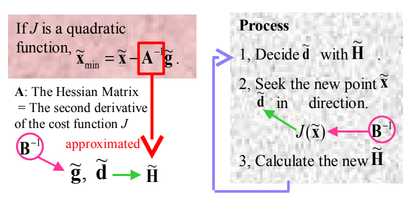
Fig.1. Schematic figure of MOVE System

Analyses of temperature and salinity fields are represented by the linear combination of T-S EOFs in MOVE System. The amplitude of the EOFs are estimated by minimizing the cost function J. Minimizing the cost function has two difficulties: nonlinearity and inversion of a large-dimensional non-diagonal matrix $\mathbf{B}$. POpULar is then applied for the minimization because it can overcome these two difficulties at the same time.

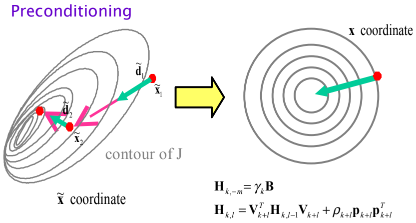
3. POpULar

POpULar is a descent scheme based on the theory of quasi-Newton method. The cost function is preconditioned with background error covariance matrix $\mathbf{B}$ and the linear part of the cost function is calculated with recursive equations without the inversion of $\mathbf{B}$.

Quasi-Newton method



Preconditioning



$$\tilde{\mathbf{H}}_{k,-m} = \gamma_k\mathbf{I}$$
$$\tilde{\mathbf{H}}_{k,l} = \tilde{\mathbf{V}}_{k+l}^T\tilde{\mathbf{H}}_{k,l-1}\tilde{\mathbf{V}}_{k+l} + \tilde{\rho}_{k+l}\tilde{\mathbf{p}}_{k+l}\tilde{\mathbf{p}}_{k+l}^T$$
$$\tilde{\mathbf{d}}_k = -\tilde{\mathbf{H}}_{k,0}\tilde{\mathbf{g}}_k$$

where

$$\tilde{\rho}_k = 1/\tilde{\mathbf{y}}_k^T\tilde{\mathbf{p}}_k$$
$$\tilde{\mathbf{V}}_k = \mathbf{I} - \rho_k\tilde{\mathbf{y}}_k\tilde{\mathbf{p}}_k^T$$
$$\tilde{\mathbf{p}}_k = \tilde{\mathbf{x}}_k - \tilde{\mathbf{x}}_{k-1} = \alpha_k\tilde{\mathbf{d}}_{k-1}$$
$$\tilde{\mathbf{y}}_k = \tilde{\mathbf{g}}_k - \tilde{\mathbf{g}}_{k-1}$$

$$\mathbf{H}_{k,-m} = \gamma_k\mathbf{B}$$
$$\mathbf{H}_{k,l} = \mathbf{V}_{k+l}^T\mathbf{H}_{k,l-1}\mathbf{V}_{k+l} + \rho_{k+l}\mathbf{p}_{k+l}\mathbf{p}_{k+l}^T$$
$$\mathbf{d}_k = -\mathbf{H}_{k,0}\mathbf{g}_k$$

where

$$\tilde{\mathbf{x}} = \left(\sqrt{\mathbf{B}}\right)^{-1}\mathbf{x}$$
$$\tilde{\mathbf{d}} = \left(\sqrt{\mathbf{B}}\right)^{-1}\mathbf{d}$$
$$\tilde{\mathbf{g}} = \sqrt{\mathbf{B}}\mathbf{g}$$
$$\mathbf{H}_{k,l} = \sqrt{\mathbf{B}}\tilde{\mathbf{H}}_{k,l}\sqrt{\mathbf{B}}$$

$$\mathbf{d}_k = -\gamma_k\mathbf{h}_k + \sum_{l=k-m+1}^{k}(a_{z,k,l}\mathbf{z}_l + a_{p,k,l}\mathbf{p}_l)$$

where $\mathbf{h}_k = \mathbf{B}\mathbf{g}_k$, $\mathbf{z}_k = \mathbf{B}\mathbf{y}_k = \mathbf{h}_k - \mathbf{h}_{k-1}$

$\times\mathbf{B}^{-1}$ →

Dividing the cost function

$$J(x_k) = \boxed{\frac{1}{2}\mathbf{x}_k^T\mathbf{B}^{-1}\mathbf{x}_k} + J_{nl}(x_k) \rightarrow K_k \qquad \mathbf{g}_k = \boxed{\mathbf{B}^{-1}\mathbf{x}_k} + \mathbf{g}_{nl}(\mathbf{x}_k) \rightarrow \mathbf{c}_k$$

Define $\mathbf{e}_k = \mathbf{B}^{-1}\mathbf{d}_k$, $\mathbf{c}_k = \mathbf{B}^{-1}\mathbf{x}_k$, $K_k = \frac{1}{2}\mathbf{x}_k^T\mathbf{B}^{-1}\mathbf{x}$

$$\mathbf{c}_k = \mathbf{c}_{k-1} + \alpha_k\mathbf{e}_{k-1} \xleftarrow{\times\mathbf{B}^{-1}} \mathbf{x}_k = \mathbf{x}_{k-1} + \alpha_k\mathbf{d}_{k-1}$$

$$K_k = K_{k-1} + \alpha_k\mathbf{d}_{k-1}^T(\mathbf{c}_{k-1} + \frac{\alpha_k}{2}\mathbf{e}_{k-1})$$

$$\mathbf{e}_k = -\gamma_k\mathbf{g}_k + \sum_{l=k-m+1}^{k}(a_{z,k,l}\mathbf{y}_l + a_{p,k,l}\mathbf{q}_l)$$

where $\mathbf{q}_k = \mathbf{B}^{-1}\mathbf{p}_k = \alpha_k\mathbf{e}_{k-1}$

4. Nonlinear constraints in 3DVAR MOVE

Nonlinear constraints had been avoided in 3DVAR ocean analyses before because it is difficult to handle the nonlinearity and the horizontal correlation of increments together with a conventional descent scheme (ex., the scheme of Derber and Rosati, 1989). POpULar does, however, allow us to introduce nonlinear constraints easily in 3DVAR MOVE System.

Sea surface topography

Sea Surface Dynamic Height (SSDH) is calculated from the analyzed T, S for the comparison with satellite altimetry data. There is a nonlinearity between T/S and SSDH through the density equation. Considering this nonlinearity improves analysis in MOVE System. For example, using linearized equation tends to estimate warm eddy too warm. This error is reduced by using the precise nonlinear equation.

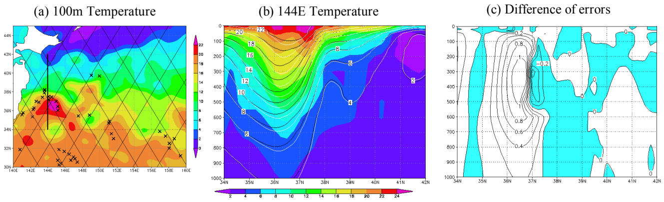
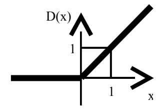


Fig. 2. (a) 100m-depth temperature analysis with nonlinearity. Crosses show the points of observation (ARGO floats). (b) 144E vertical section of temperature fields. Color: observation, black: nonlinear, white: linear. (c) the difference of absolute temperature errores ([linear] – [nonlinear]). No shade means nonlinear estimation is better.

Constraint avoiding the density inversion

$$\alpha = \frac{1}{a_1^2}\sum_p\sum_k\left[D(\rho_{p,k} - \rho_{p,k-1})\right]^2$$



α is a constraint avoiding density inversion because it has a large value when ρ_k-ρ_{k-1} is positive.

It is difficult to express the exceptionally deepening of the thermocline by EOF combinations. A false temperature maximum tends to be analyzed above the thermocline, instead. This shortcoming is improved by introducing the constraint above.

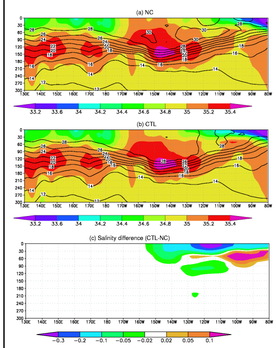


Fig. 3. Temperature (contour) and salinity (color) fields of the section along the equator in November, 1997. (a) using no constraint, (b) using the constraint , (c) difference of salinity field .

Variational QC

$$J_o = \frac{1}{2}\mathcal{Q}(\mathcal{H}(\mathbf{w})-\mathbf{y})^T\mathbf{R}^{-1}\mathcal{Q}(\mathcal{H}(\mathbf{w})-\mathbf{y})$$

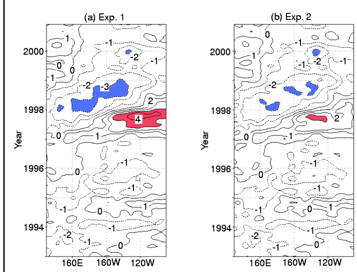
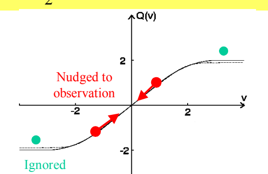


Fig. 4. Longitude and time section of the heat content anomaly at the equator. (a) with variational QC. (b) with the conventional 3σ check.



Data of exceptional large anomaly in El Nino periods is discarded by conventional 3σ check. Variational QC prevents those data for being discarded because variational QC compares the observation data not with the background value like the conventional QC, but with the analyzed value.

5. 4DVAR MOVE with horizontal correlation of analysis increments

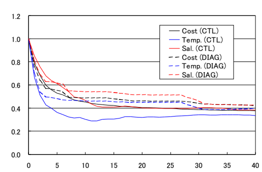


Fig. 5. Plots of costs and RMSE of T, S in initial state normalized by the Guess.

4DVAR MOVE applies the OGCM codes in the observation operator. The horizontal correlation of analysis increments had been ignored in 4DVARs before because it is difficult to handle with the nonlinear observation operator. POpULar allows us to resolve the problem in MOVE System. We implemented the two identical twin experiments with and without the correlation (CTL and DIAG, respectively). Better estimation is acquired from CTL. It means that considering the horizontal correlation has a positive impact on 4DVAR systems.

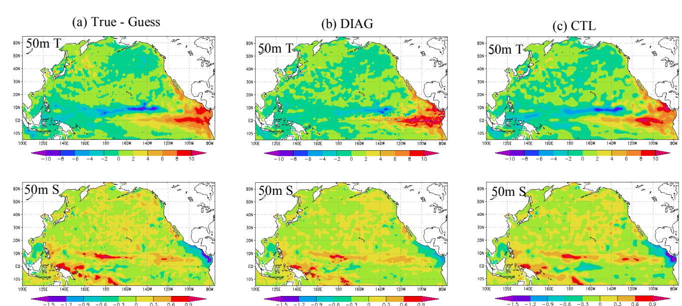


Fig. 6. Comparison of analysis increments in 50m T, S in initial state.

6. Summary

MOVE System allows us to apply nonlinear constraints with the non-diagonal background error covariance matrix in the variational analysis scheme in MOVE System. 3DVAR MOVE is sophisticated by applying nonlinear constraints for employing altimetry data, avoiding density inversion, and a variational QC procedure. We also confirmed that considering horizontal correlation among analysis increments reduces the analysis error in a 4DVAR identical twin experiment.

7. Papers about this research

Fujii, Y., 2004: Preconditioned Optimizing Utility for Large-dimensional analyses (PopULar). *J. Oceanogr.*, accepted.

Fujii, Y. and M. Kamachi, 2003a: Three-dimensional analysis of temperature and Salinity in the equatorial Pacific using a variational method with vertical coupled temperature-Salinity empirical orthogonal function modes. *J. Geophys. Res.*, **108**(C9), 3217, doi:10.1029/2002JC001745.

Fujii Y. and M. Kamachi, 2003b: A nonlinear preconditioned quasi-Newton method without inversion of a first-guess covariance matrix in variational analyses. *Tellus*, **55A**, 450-454.

Fujii, Y., S. Ishizaki, and M. Kamachi, 2004: Application of nonlinear constraints in a three-dimensional variational ocean analysis. *J. Oceanogr.*, accepted.